```
> restart; kernelopts(version); Digits:=18;
  read cat(myLib, "nearest.mpl"):
```

Maple 12.02, IBM INTEL NT, Dec 10 2008 Build ID 377066

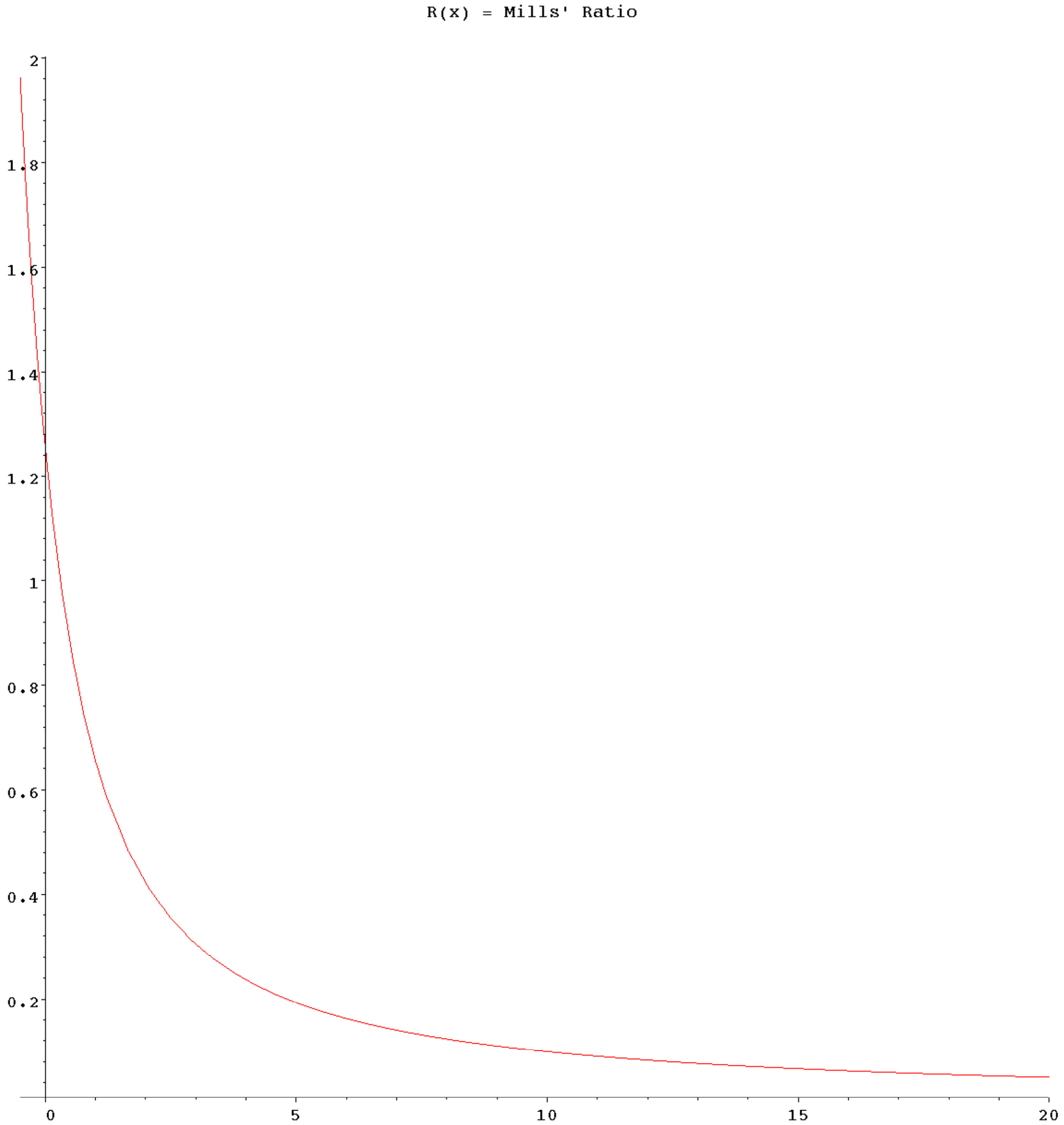
Digits := 18

```
> R:= x -> (1-N(x))/D(N)(x); simplify(R(x)): convert(%,erfc):
  R:=unapply(%,x);

  plot(R(x), x=-0.5..20, myPlotDefault, title="R(x) = Mills' Ratio");
```

$$R := x \to \frac{1 - N(x)}{D(N)(x)}$$

$$R := x \to \frac{1}{2}\operatorname{erfc}\left(\frac{1}{2}\,x\,\sqrt{2}\right)\sqrt{\pi}\,\sqrt{2}\,\mathbf{e}^{(1/2\,x^2)}$$




```
> # calling Mills Ratio from a DLL
  currentdir(): theDLL:=cat(%,`\\BS_strict.dll`):

  R_DLL:=define_external(
    'my_MillsRatio', 'C',
    'x'::float[8],
    'RETURN'::float[8],
    LIB=theDLL);
```

R_DLL := **proc**(x::numeric)

option call_external,

define_external(my_MillsRatio, C, x::float[8], RETURN::float[8], LIB = "D:_Work\\vc2005\\dir_Finance\\BS_strict\\release\\BS_strict.dll");

call_external(Array(1..8, [...], datatype = integer[4], readonly), false, args)

end proc

```
> oldDigits:=Digits: Digits:=60:
  R_DLL(0);
  evalf(R(0));
  `absolute error in x=0` = % - %%: evalf[18](%);
  Digits:=oldDigits:
```

1.25331413731550034

1.25331413731550025120788264240552262650349337030496915831496

absolute error in x=0 = -0.887921173575944774 10^{-16}

```
> errorAbs_IEEE:= proc(X)
    local x,rho;
    Digits:=36;
    x:= eval(nearest(evalf(X)));
    evalf(R(x)-R_DLL(x));
    #eval(nearest(%)); %/2^(-52); 16*evalf(%); round(%)/16;
  end proc;
```

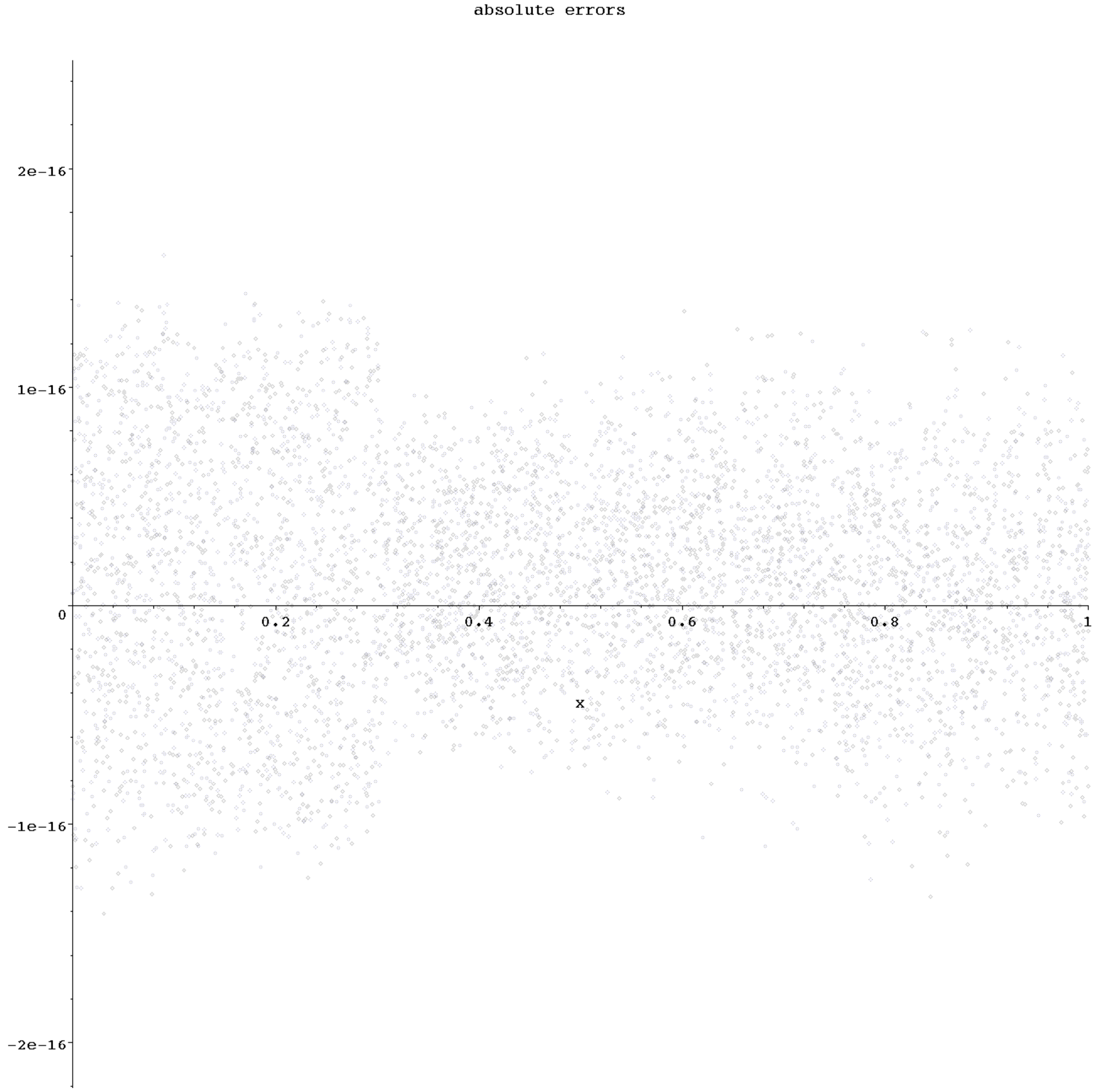
errorAbs_IEEE := **proc**(X) **local** x, ρ; Digits := 36; x := eval(nearest(evalf(X))); evalf(R(x) – R_DLL(x)) **end proc**

```
> '`absolute error`[x=0] = errorAbs_IEEE(0)'; %;
```

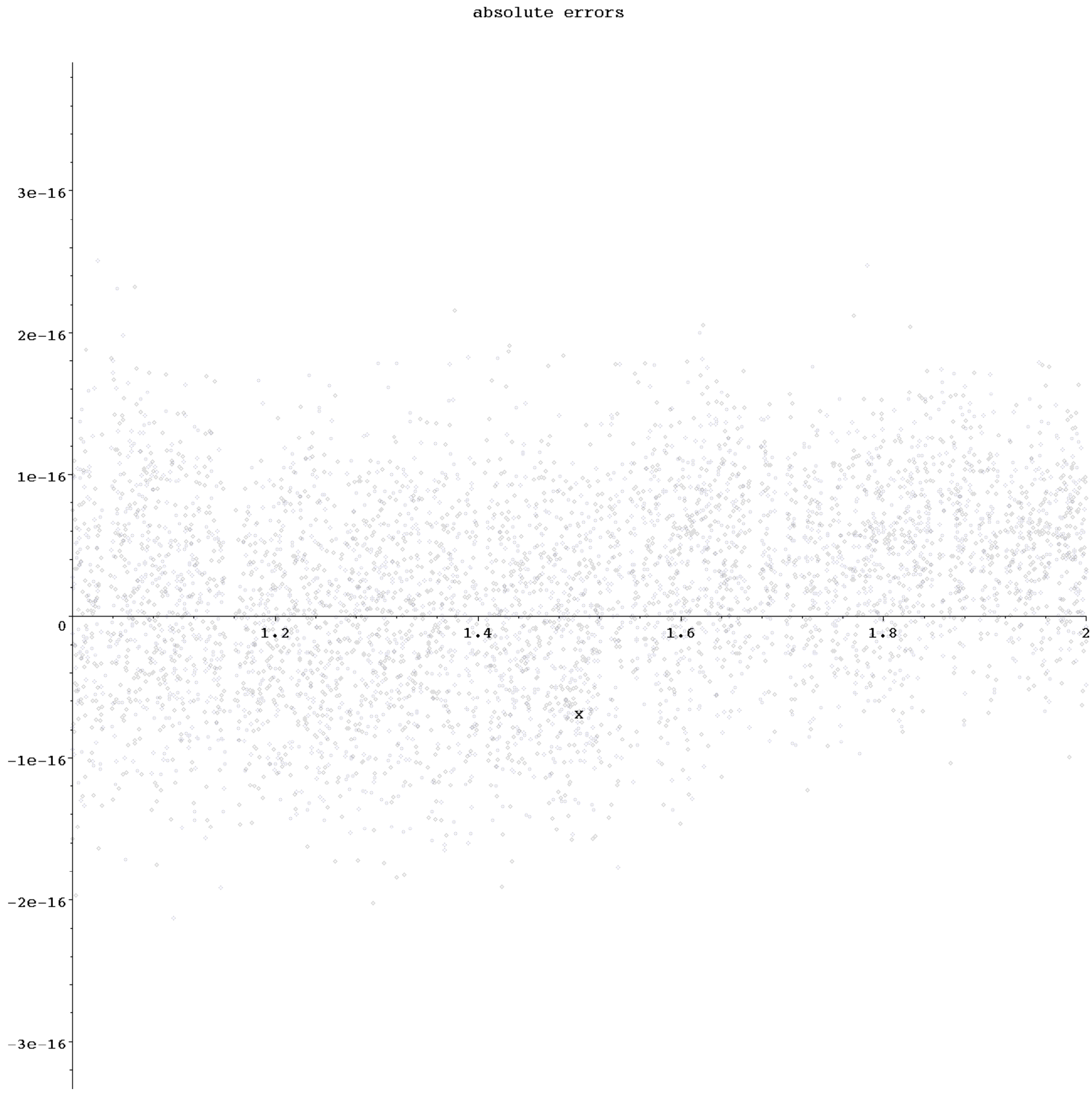
$\text{absolute error}_{x=0} = \text{errorAbs\_IEEE}(0)$

$\text{absolute error}_{x=0} = -0.8879211735759447738\ 10^{-16}$

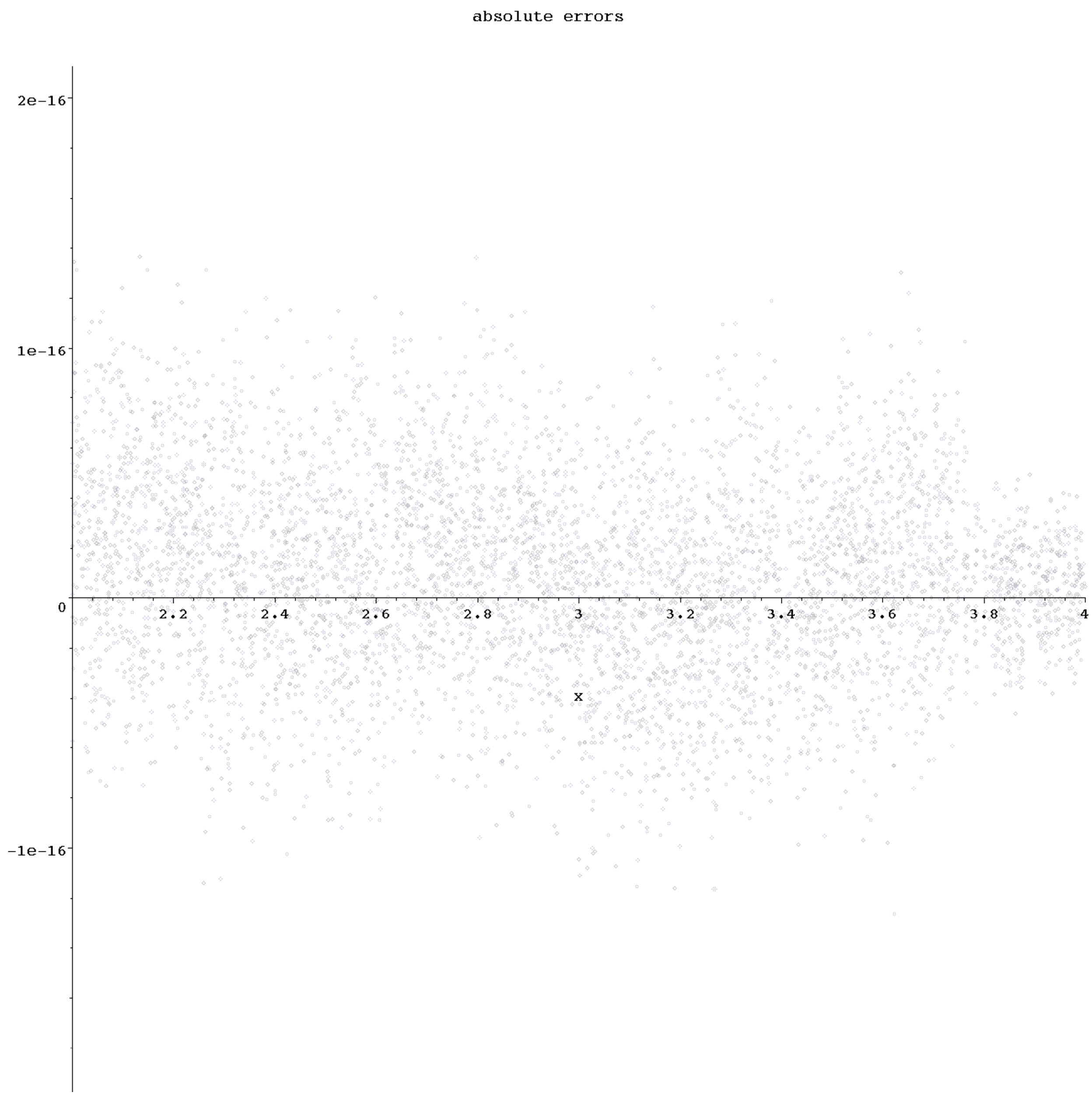
```
>
> plot('errorAbs_IEEE'(x),    x=0..1, numpoints=200, style=point, myPlotDefault, title="absolute
  errors", color=grey);
```

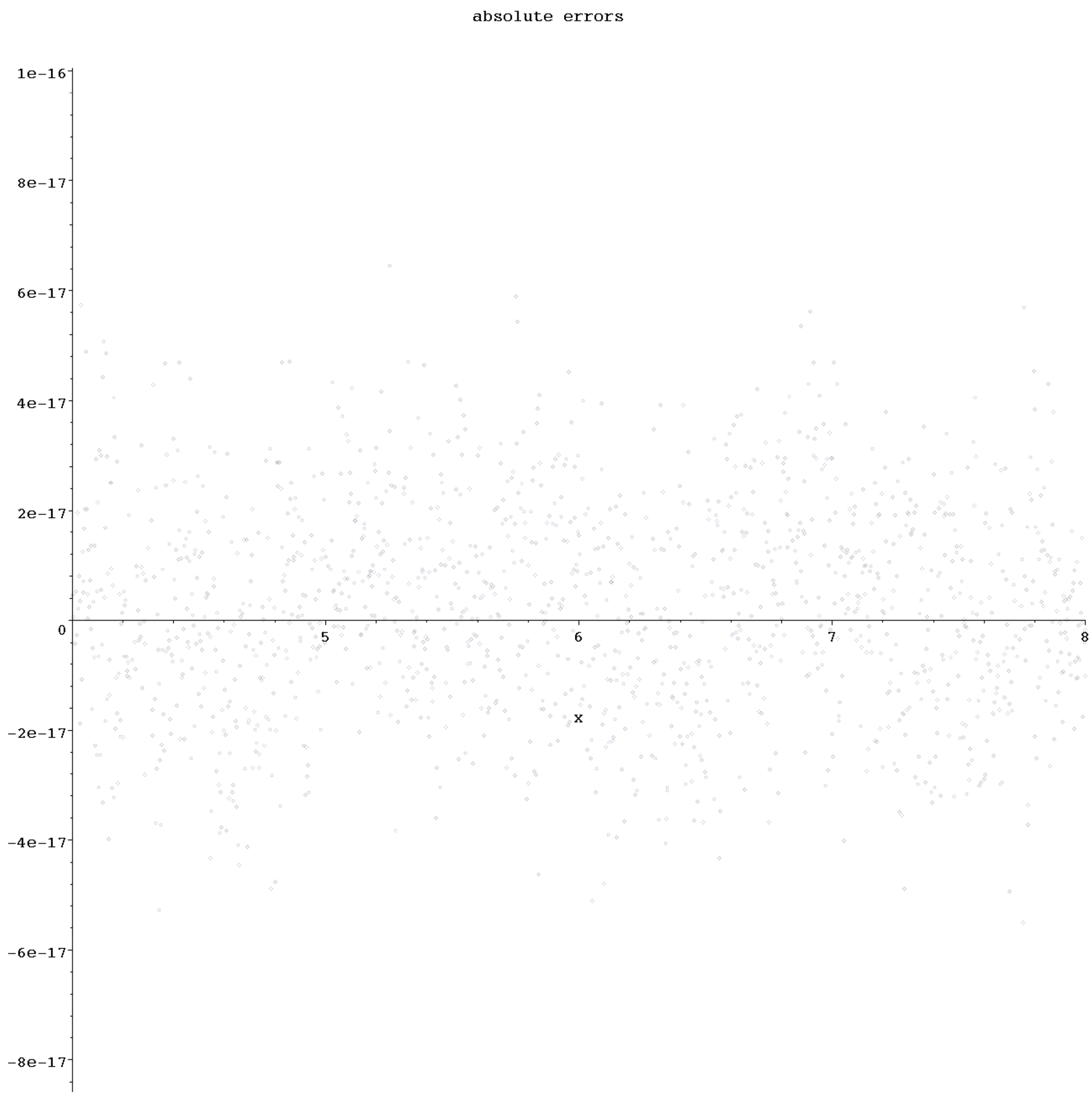



```
> plot('errorAbs_IEEE'(x),    x=1..2, numpoints=200, style=point, myPlotDefault, title="absolute
  errors", color=grey);
```

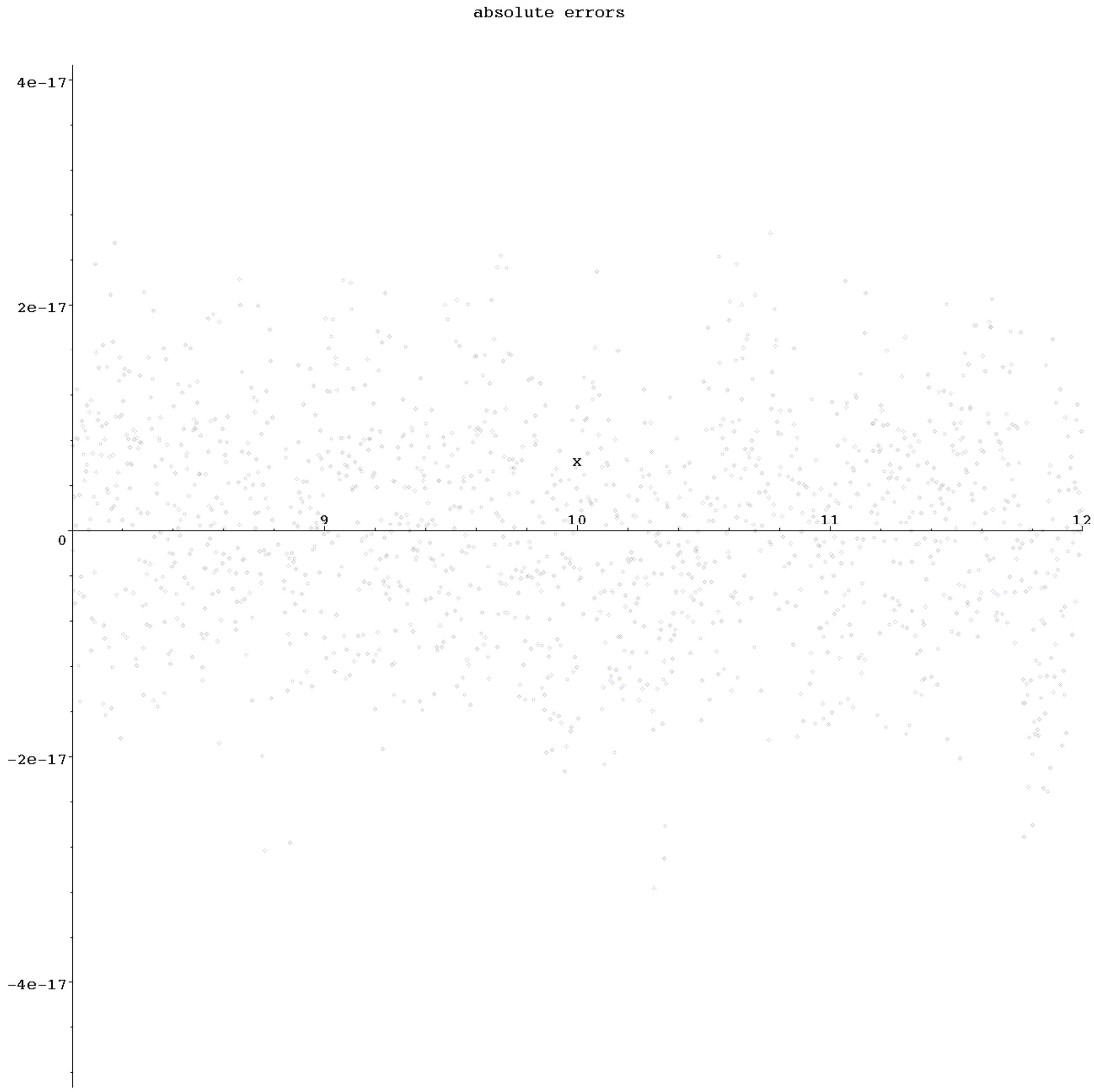



```
>
> plot('errorAbs_IEEE'(x), x=2..4, numpoints=200, style=point, myPlotDefault, title="absolute errors",
  color=grey);
  plot('errorAbs_IEEE'(x), x=4..8, style=point, myPlotDefault, title="absolute errors", color=grey);
  plot('errorAbs_IEEE'(x), x=8..12, style=point, myPlotDefault, title="absolute errors", color=grey);
```

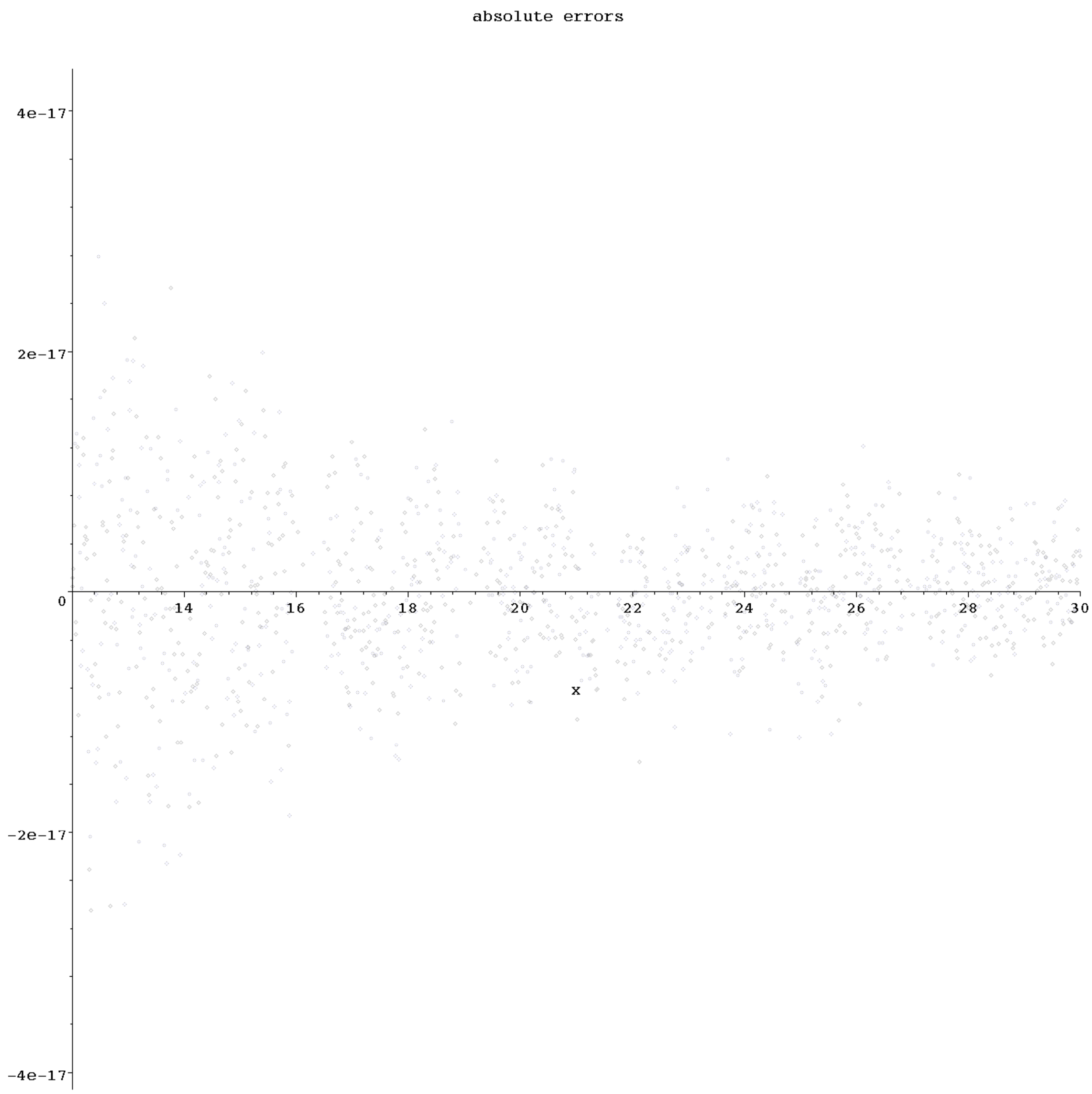

absolute errors
2e-16
1e-16
0
-1e-16
2.2
2.4
2.6
2.8
3
3.2
3.4
3.6
3.8
4
x

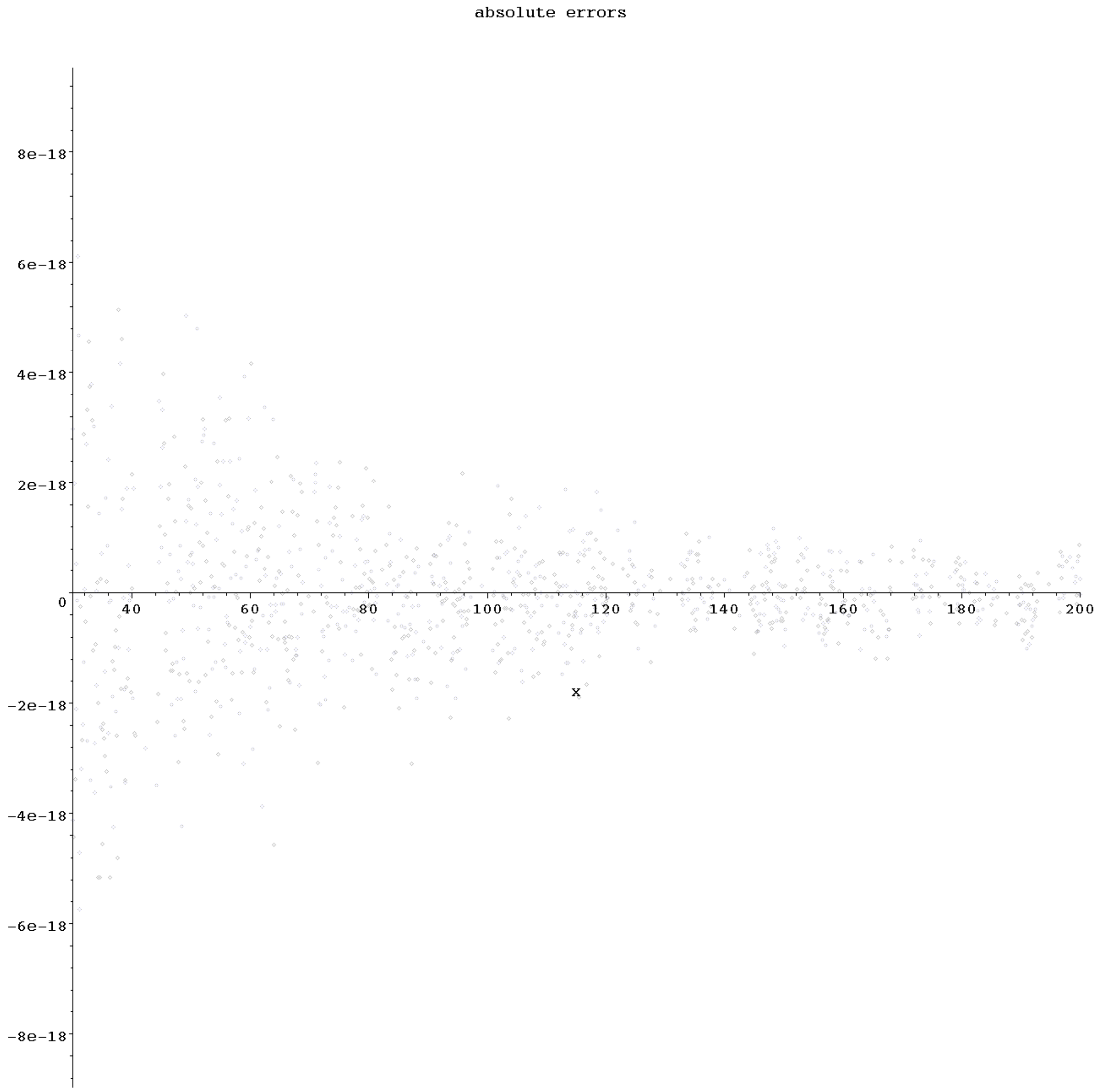


absolute errors
1e-16
8e-17
6e-17
4e-17
2e-17
0
-2e-17
-4e-17
-6e-17
-8e-17
5
6
7
8
x




```
> plot('errorAbs_IEEE'(x), x=12..30, style=point, myPlotDefault, title="absolute errors", color=grey);
  plot('errorAbs_IEEE'(x), x=30..200, style=point, myPlotDefault, title="absolute errors", color=grey);
```


absolute errors
4e-17
2e-17
0
-2e-17
-4e-17
14
16
18
20
22
24
26
28
30
x




```
> oldDigits:=Digits: Digits:=10000;
  xTst:=16000;
  'ln(R(a))'; simplify(%,symbolic): convert(%,erfc);
  subs(a=xTst,%);  evalf(%):
  Digits:=oldDigits;
  evalf(%%); exp(%);
  R_DLL(xTst); %-%%;
```

$$Digits := 10000$$

$$xTst := 16000$$

$$\ln(R(a))$$

$$-\frac{1}{2}\ln(2)+\frac{1}{2}\ln(\pi)+\ln\left(\operatorname{erfc}\left(\frac{a\sqrt{2}}{2}\right)\right)+\frac{a^2}{2}$$

$$-\frac{1}{2}\ln(2)+\frac{1}{2}\ln(\pi)+\ln(\operatorname{erfc}(8000\sqrt{2}))+128000000$$

$$Digits := 18$$

$$-9.68034400512816825$$

$$0.0000624999997558593780$$

$$0.0000624999998445752560$$

$$0.887158780\ 10^{-13}$$

```
>
```

relative errors as multiples of DBL_EPSILON

```
> 'pow(2,-52)': '%'=evalf(%);
```

$$pow(2, -52) = 0.222044604925031308\ 10^{-15}$$

```
> errorRel_IEEE:= proc(X)
  local x,rho;
  Digits:=36;
  x:= eval(nearest(evalf(X)));
  evalf(1 - R_DLL(x)/R(x));
  eval(nearest(%)); %/2^(-52); 16*evalf(%); round(%)/16;
  end proc;
```

errorRel_IEEE := **proc**(X)

local x, ρ;

Digits := 36;

x := eval(nearest(evalf(X)));

evalf(1 − R_DLL(x) / R(x));

eval(nearest(%));

%*4503599627370496;

16*evalf(%);

1 / 16*round(%)

end proc

```
> '`relative error`[x=0] = errorRel_IEEE(0)*epsilon'; %;
  '`relative error`[x=1] = errorRel_IEEE(1)*epsilon'; %;
```

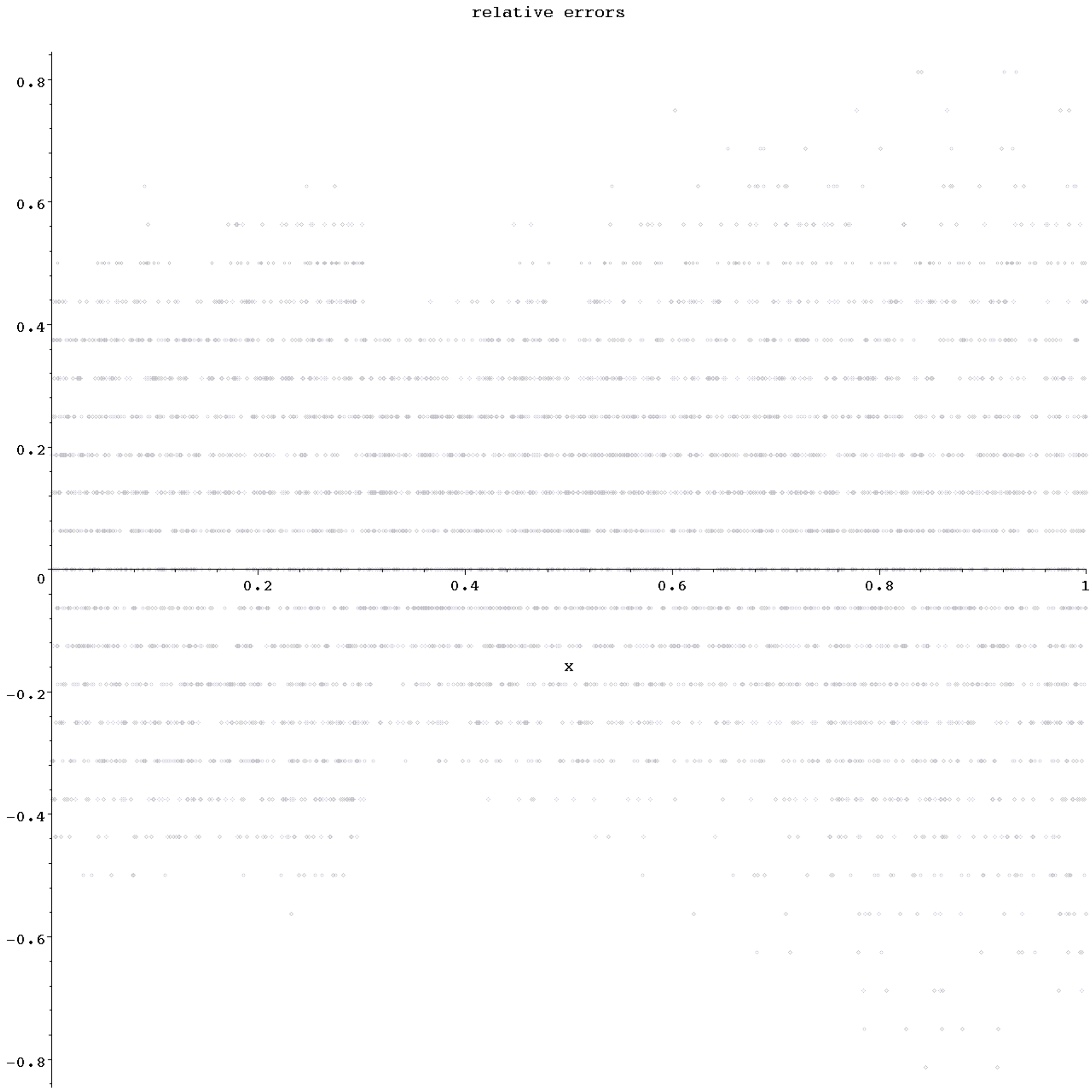
$$\text{relative error}_{x=0} = \text{errorRel\_IEEE}(0)\ \varepsilon$$

$$\text{relative error}_{x=0} = -\frac{5\ \varepsilon}{16}$$

$$\text{relative error}_{x=1} = \text{errorRel\_IEEE}(1)\ \varepsilon$$

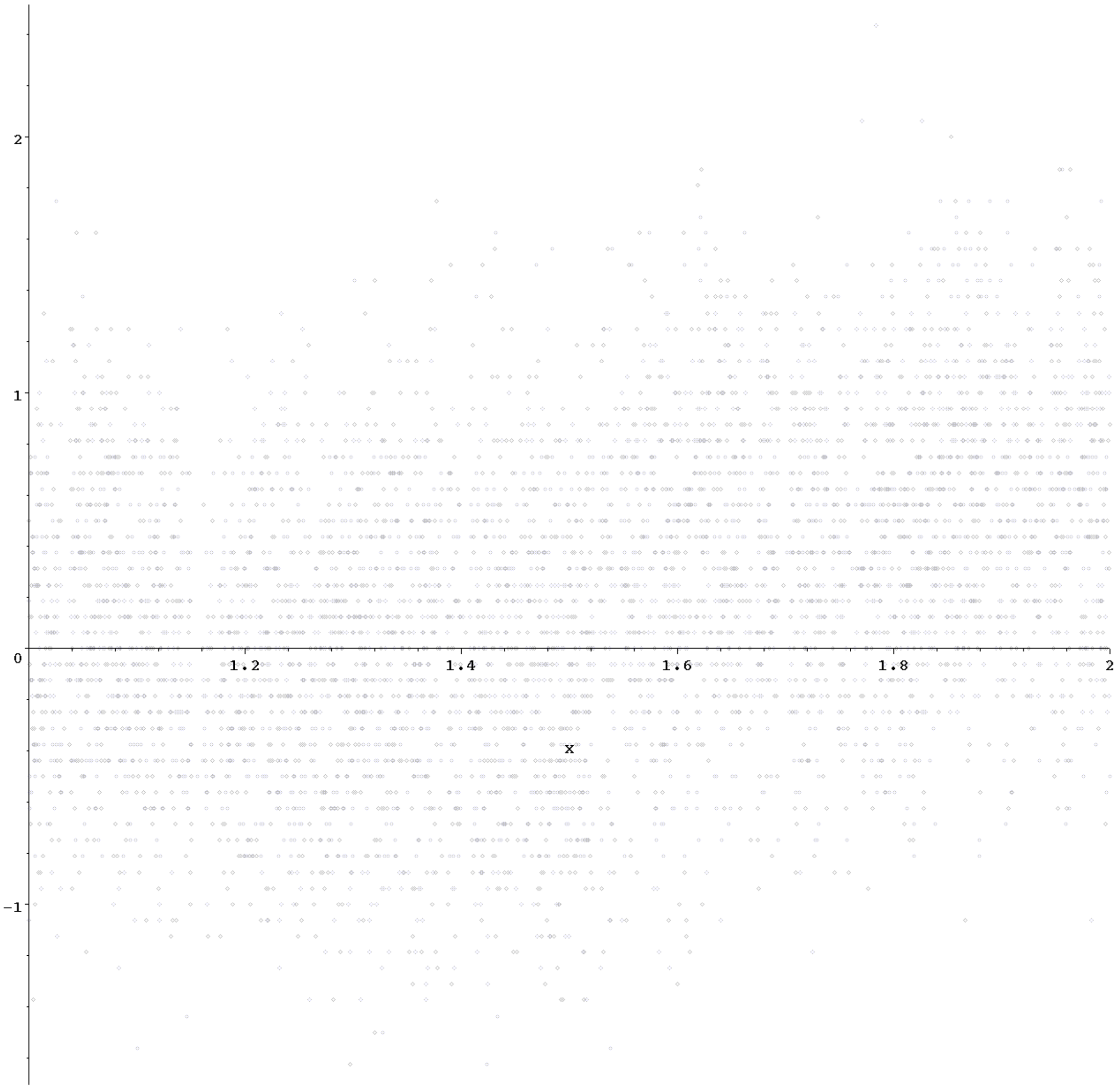
$$\text{relative error}_{x=1} = \frac{3\ \varepsilon}{16}$$

```
>
> plot('errorRel_IEEE'(x),    x=0..1, numpoints=200, style=point, myPlotDefault, title="relative
  errors", color=grey);
```

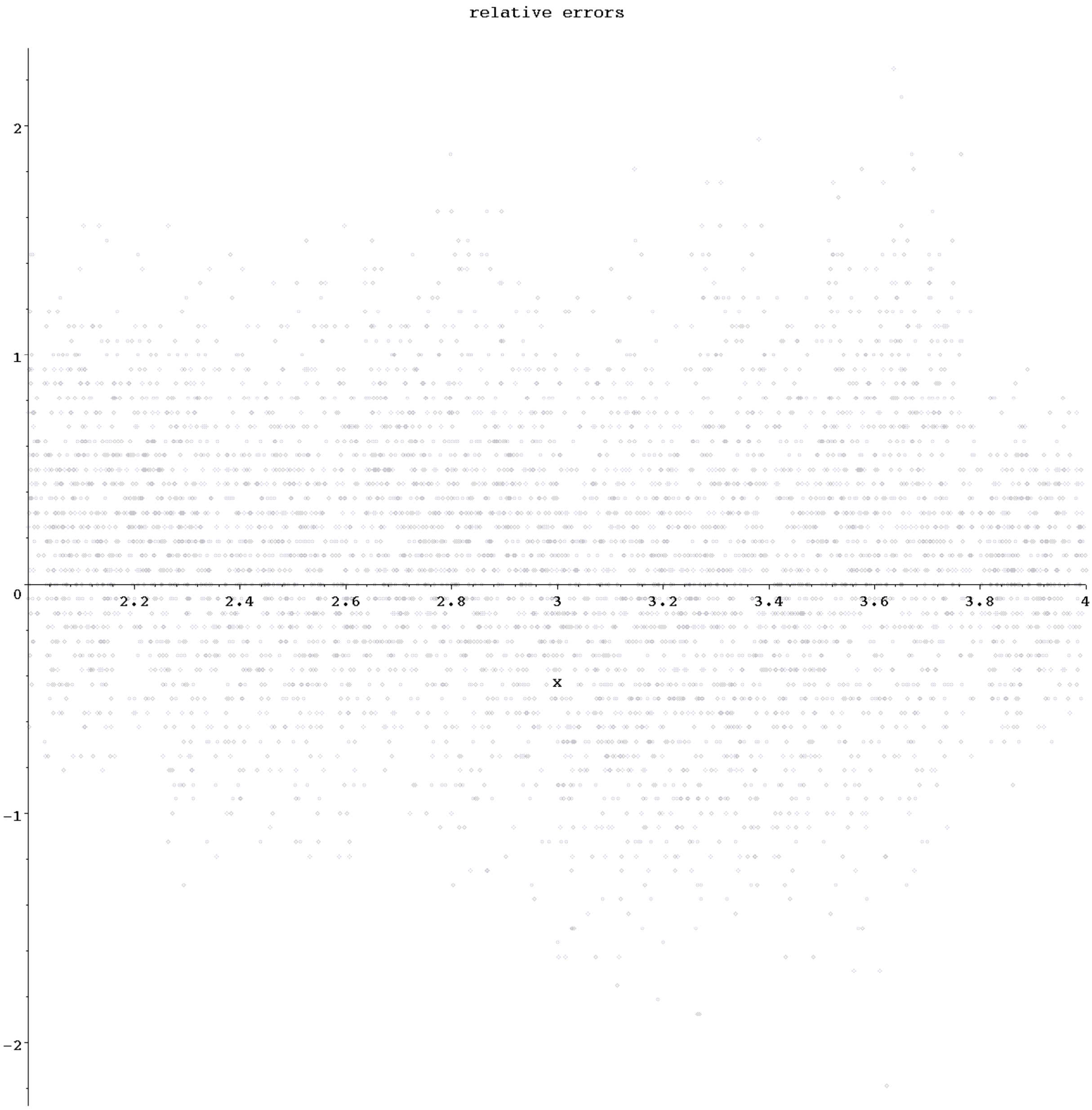


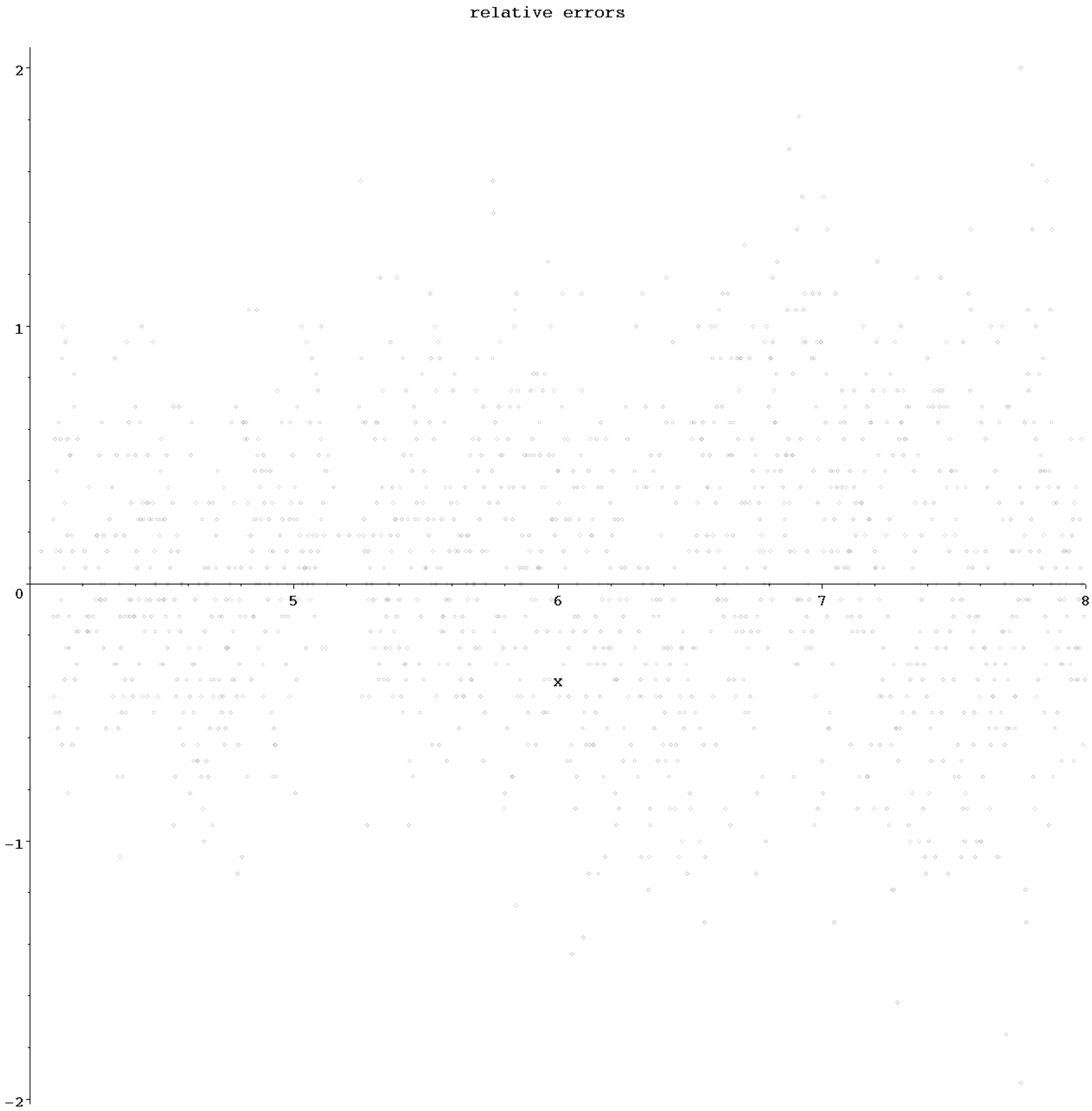

```
> plot('errorRel_IEEE'(x),    x=1..2, numpoints=200, style=point, myPlotDefault, title="relative
  errors", color=grey);
```

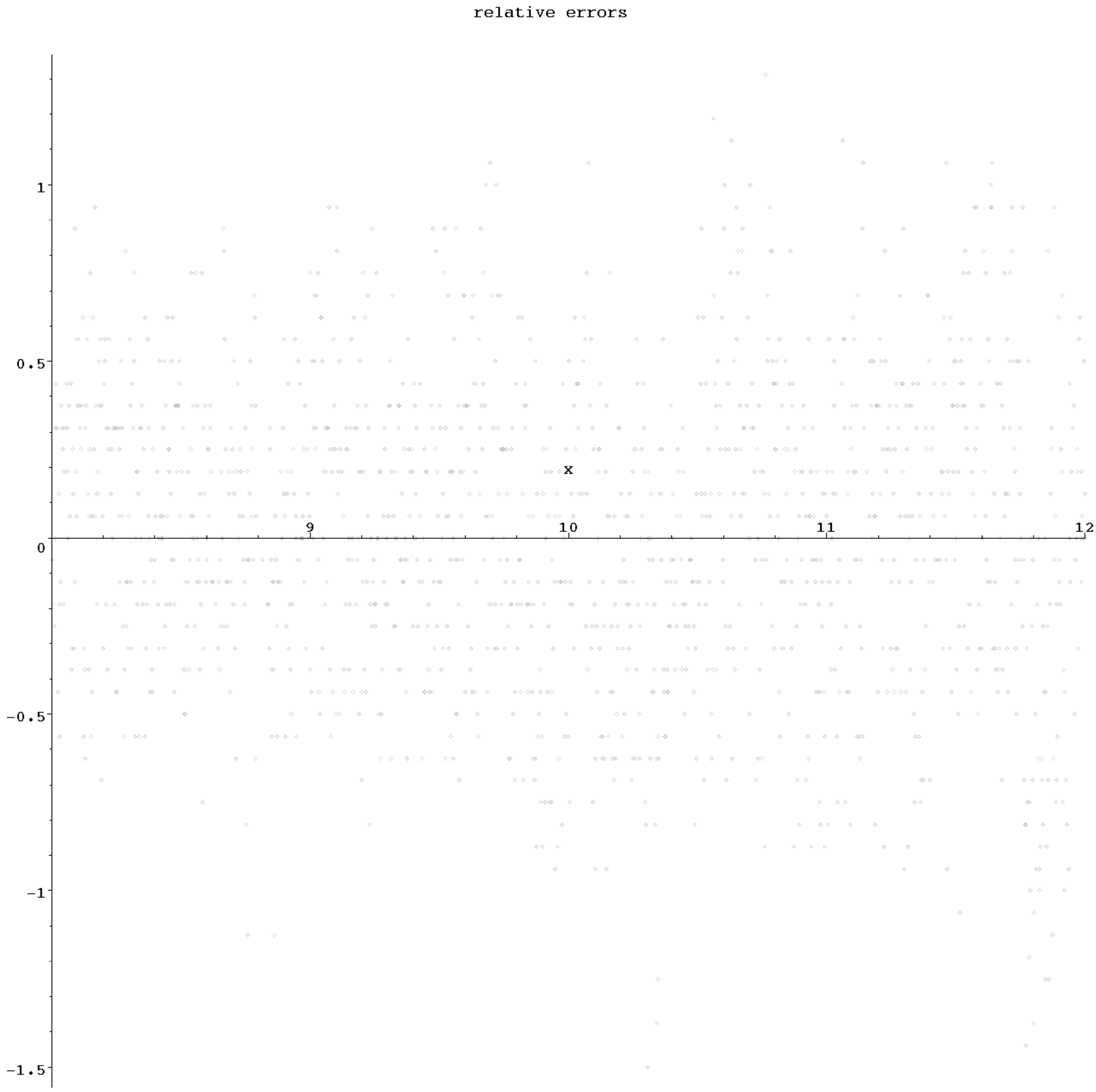
relative errors



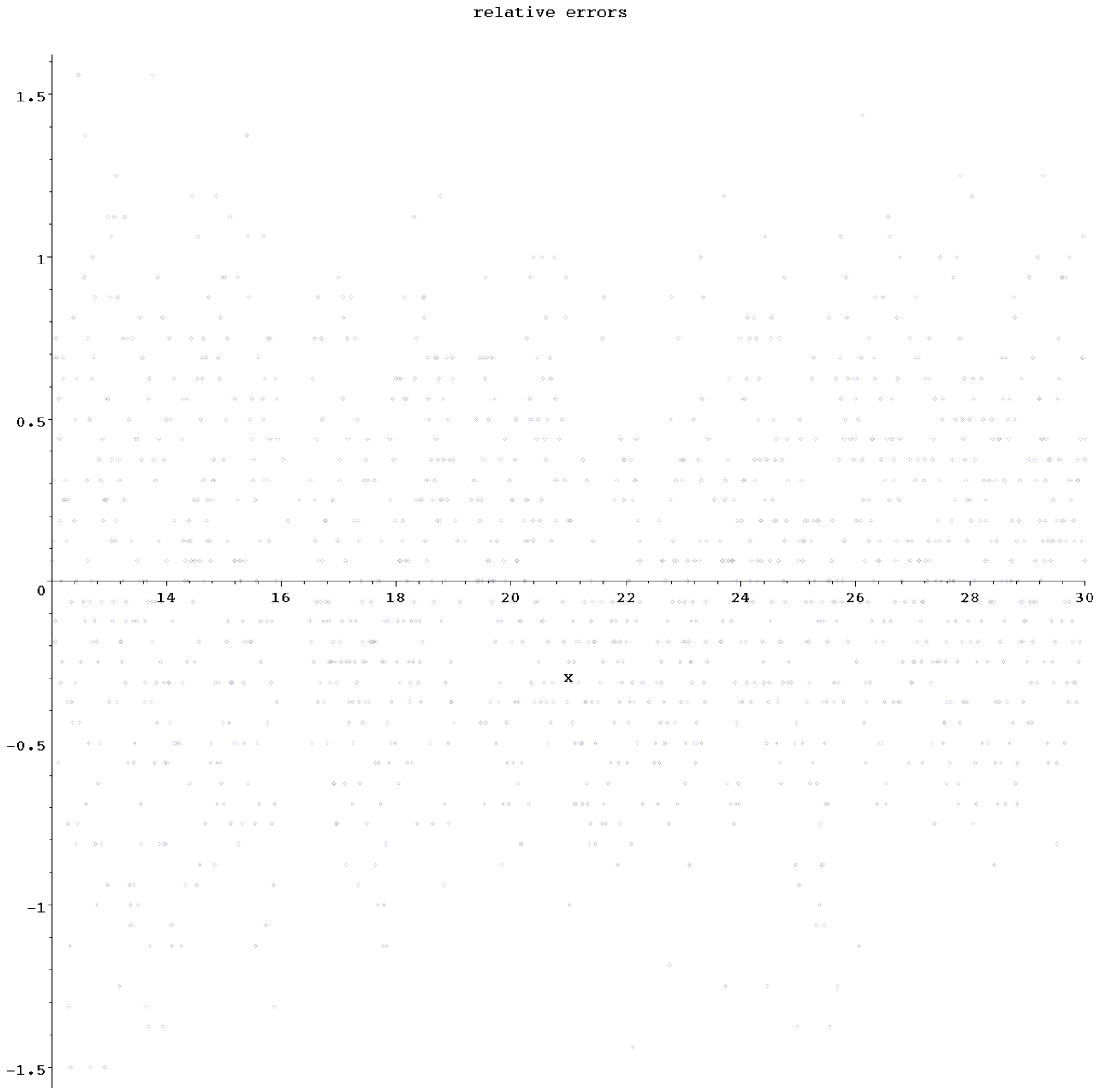

```
> 
> plot('errorRel_IEEE'(x), x=2..4, numpoints=200, style=point, myPlotDefault, title="relative
  errors", color=grey);
  plot('errorRel_IEEE'(x), x=4..8, style=point, myPlotDefault, title="relative errors", color=grey);
  plot('errorRel_IEEE'(x), x=8..12, style=point, myPlotDefault, title="relative errors", color=grey);
```

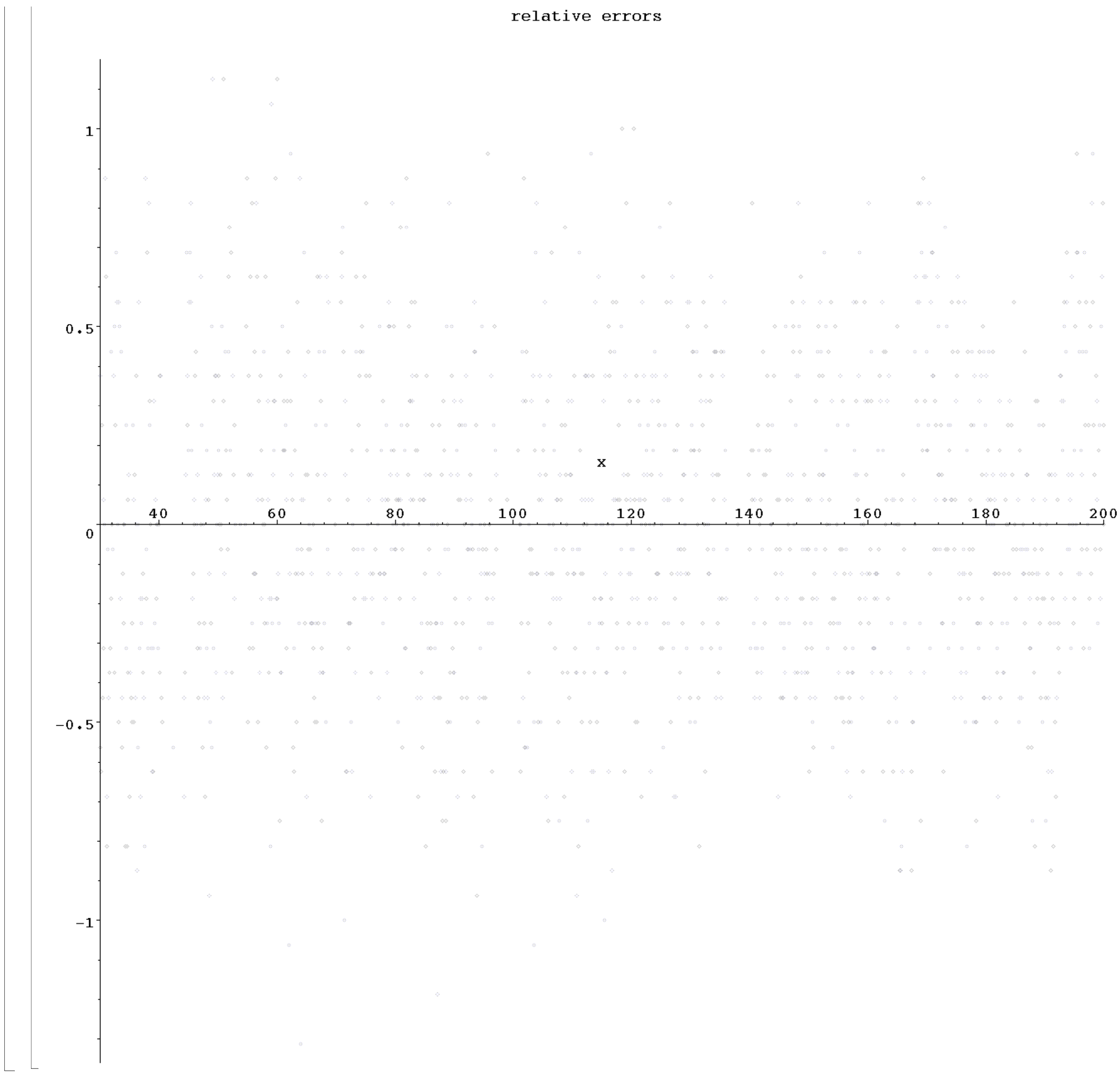

relative errors
2
1
0
−1
−2
2.2
2.4
2.6
2.8
3
3.2
3.4
3.6
3.8
4
x


relative errors
2
1
0
−1
−2
5
6
7
8
x




```
> plot('errorRel_IEEE'(x), x=12..30, style=point, myPlotDefault, title="relative errors",
  color=grey);
  plot('errorRel_IEEE'(x), x=30..200, style=point, myPlotDefault, title="relative errors",
  color=grey);
```


relative errors
1.5
1
0.5
0
−0.5
−1
−1.5
14
16
18
20
22
24
26
28
30
x


relative errors
1
0.5
0
-0.5
-1
40
60
80
100
120
140
160
180
200
x