

```
[ > restart;
  > Digits:=14;
```

Digits := 14

MCC Madan, Carr, Chang 1998

```
> 'N(a/sqrt(u)+b*sqrt(u))*u^(c-1)*exp(-u)';
Int(%,u=0..infinity)/GAMMA(c): #value(%):
M:= unapply(%,a,b,c); #assume(0<c): assume(a::real): assume(b::real):
```

$$N\left(\frac{a}{\sqrt{u}} + b\sqrt{u}\right) u^{(c-1)} e^{-u}$$

$$M := (a, b, c) \rightarrow \frac{1}{\Gamma(c)} \int_0^{\infty} N\left(\frac{a}{\sqrt{u}} + b\sqrt{u}\right) u^{(c-1)} e^{-u} du$$

```
[ that function is called  $\Psi(a, b, \gamma)$  in the paper (appendix).
```

```
> N:= x -> 1/2+1/2*erf(1/2*x*2^(1/2));
```

$$N := x \rightarrow \frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{1}{2} x \sqrt{2}\right)$$

```
> #assume(0<sigma): assume(nu::real): assume(theta::real):
```

```
zeta:=-theta/sigma^2;
s:='sigma/sqrt(1+(theta/sigma)^2*nu/2)';
```

```
alpha:='zeta*s';
c1:='nu*(alpha+s)^2/2';
c2:='nu*alpha^2/2';
```

```
d0:='1/s*(ln(S/K)+r*t+t/nu*ln((1-c1)/(1-c2)))';
```

$$\zeta := -\frac{\theta}{\sigma^2}$$

$$s := \frac{\sigma}{\sqrt{1 + \frac{\theta^2 \nu}{2 \sigma^2}}}$$

$$\alpha := \zeta s$$

$$c1 := \frac{\nu (\alpha + s)^2}{2}$$

$$c2 := \frac{\nu \alpha^2}{2}$$

$$d0 := \frac{\ln\left(\frac{S}{K}\right) + r t + \frac{t \ln\left(\frac{1-c1}{1-c2}\right)}{\nu}}{s}$$

```
> 'S*M(d0*sqrt((1-c1)/nu), (alpha+s)*sqrt(nu/(1-c1)), t/nu) -
  K*exp(-r*t)*M(d0*sqrt((1-c2)/nu), (alpha+0)*sqrt(nu/(1-c2)), t/nu)';
```

```
MCC:='unapply(%,S,K,t,r,sigma,nu,theta)'; # indets(MCC(S,K,t,r)):
indets(%,atomic): indets(%);
assume(0<S); assume(0<K); assume(0<t); assume(0<=r);
```

MCC := unapply(

$$S M\left(d0 \sqrt{\frac{1-c1}{v}}, (\alpha+s) \sqrt{\frac{v}{1-c1}}, \frac{t}{v}\right) - K e^{(-rt)} M\left(d0 \sqrt{\frac{1-c2}{v}}, \alpha \sqrt{\frac{v}{1-c2}}, \frac{t}{v}\right), S, K, t, r, \sigma, v, \theta$$

that function is called $c(S(0); K, t)$ in the paper

Examples

```
> tstData:=[S=100.0, t=1.0, r=0.02, sigma=0.1737, nu=0.679, theta=0.0492];
      tstData := [S = 100.0, t = 1.0, r = 0.02, σ = 0.1737, ν = 0.679, θ = 0.0492]
let the Call depend only on K (to have 'good' run times)
> MCC(S,K,t,r,sigma,nu,theta): eval(%,tstData):

      VGC:=unapply(%,K):;

> VGC( 90.0): evalf(%);
      VGC(100.0): evalf(%);
      VGC(110.0): evalf(%);

                                14.123021770626
                                7.477234761060
                                3.408571065263

now do it the reasonable way
> params:=[sigma=0.1737,nu=0.679,theta=0.0492];
      params := [σ = 0.1737, ν = 0.679, θ = 0.0492]
> MCC(S,K,t,r,sigma,nu,theta):
      subs(S=spot,K=strike,t=time,r=rates,%):
      eval(%,params): evalf(%):

      VGCall:=unapply(%,spot,strike,time,rates):

> for K in [90.0, 100.0, 110.0] do
      price[K]=evalf(VGCall(100.0, K, 1.0, 0.02));
      end do;

      price90.0 = 14.123021770623
      price100.0 = 7.477234761060
      price110.0 = 3.408571065264

check runtime:
> st:=time():
      noSteps:=100;
      for iStep from 0 to noSteps do
      evalf(VGCall(100.0, 100.0 -noSteps/2 + iStep, 0.5,0.00));
      end do:
      `average time in seconds`=evalf[3]((time()-st)/(noSteps+1));

                                noSteps := 100
                                average time in seconds = 0.0189
```

a pure C program may faster by a factor of 10 - 100 ...