

Digits := 18

theDLL := "D:_Work\vc2005\hyp2f1_various\hyp2f1_slim_snow\release\hyp2f1_snow.dll"

Define the functions MPL_2F1 (which is Maple's version) and hyp2f1_DLL (which is calling the DLL).

For comparing results one may wish to convert numerical inputs (decimals in Maple) to their nearest IEEE equivalent to have the same input. After that pre-processing a test routine can be feed, which gives both of the results and the relative and absolute errors.

+ prepare for testing

First: look at the 'usual critical case': points on the circle line, which are also on the diagonal (fix points at the usual linear transforms):

$$\begin{aligned} \text{tstData} &:= \left[a = \frac{1}{2}, b = \frac{1}{3}, c = \frac{1}{6}, z = e^{(1/3)I\pi} \right] \\ \text{TstData} &:= \frac{1}{2} \text{pow}(2, 0), \frac{6004799503160661}{9007199254740992} \text{pow}(2, -1), \frac{6004799503160661}{9007199254740992} \text{pow}(2, -2), \frac{1}{2} \text{pow}(2, 0) + \frac{3900231685776981}{4503599627370496} I \text{pow}(2, 0) \\ \text{DLL} &= 0.623714907712016986 + 0.958923198106978569 I \\ \text{MPL} &= 0.623714907712016986 + 0.958923198106978902 I \\ \text{error}_{\text{absolute, relative}} &= 0.333 \cdot 10^{-15}, 0.291082217408827143 \cdot 10^{-15} \end{aligned}$$

- tests

- 1

$$\begin{aligned} \text{tstData} &:= \left[a = -3\pi, b = \sqrt{2}, c = e, z = \frac{1}{e^\pi} + 1 - 0.1 \cdot 10^{-289} I \right] \\ \text{TstData} &:= \frac{2652839157010665}{4503599627370496} \text{pow}(2, 4), \frac{6369051672525773}{9007199254740992} \text{pow}(2, 1), \frac{6121026514868073}{9007199254740992} \text{pow}(2, 2), \\ &\frac{587277226695055}{112589906842624} \text{pow}(2, 1) - \frac{3511119404027961}{4503599627370496} I \text{pow}(2, -963) \\ \text{DLL} &= 0.0560182538639228261 - 0.170002900645727095 \cdot 10^{-15} I \\ \text{MPL} &= 0.0560182538639227706 - 0.172417988082930188 \cdot 10^{-15} I \\ \text{error}_{\text{absolute, relative}} &= 0.555525215208935066 \cdot 10^{-16}, 0.990938286865449774 \cdot 10^{-15} \end{aligned}$$

- 2

$$\begin{aligned} \text{tstData} &:= [a = 1, b = 1, c = 2.2, z = 1.9] \\ \text{TstData} &:= \frac{1}{2} \text{pow}(2, 1), \frac{1}{2} \text{pow}(2, 1), \frac{2476979795053773}{4503599627370496} \text{pow}(2, 2), \frac{4278419646001971}{4503599627370496} \text{pow}(2, 1) \\ \text{DLL} &= 0.315767495606635251 - 1.70873729899199533 I \\ \text{MPL} &= 0.315767495606590731 - 1.70873729899200000 I \\ \text{error}_{\text{absolute, relative}} &= 0.447642636485846822 \cdot 10^{-13}, 0.257608723959508586 \cdot 10^{-13} \end{aligned}$$

- 3

$$\begin{aligned} \text{tstData} &:= [a = 1.756, b = 4.76, c = 2.20, z = 0.85] \\ \text{TstData} &:= \frac{7908320945662591}{9007199254740992} \text{pow}(2, 1), \frac{2679641778285445}{4503599627370496} \text{pow}(2, 3), \frac{2476979795053773}{4503599627370496} \text{pow}(2, 2), \frac{7656119366529843}{9007199254740992} \text{pow}(2, 0) \\ \text{DLL} &= 2443.22685352899316 \\ \text{MPL} &= 2443.22685352899134 \\ \text{error}_{\text{absolute, relative}} &= 0.182 \cdot 10^{-11}, 0.74 \cdot 10^{-15} \end{aligned}$$

- 4

$$\begin{aligned} \text{tstData} &:= [a = 1.756, b = 4.76, c = 2.20, z = 1.95 I] \\ \text{TstData} &:= \frac{7908320945662591}{9007199254740992} \text{pow}(2, 1), \frac{2679641778285445}{4503599627370496} \text{pow}(2, 3), \frac{2476979795053773}{4503599627370496} \text{pow}(2, 2), \frac{8782019273372467}{9007199254740992} I \text{pow}(2, 1) \\ \text{DLL} &= -0.0291190062043906323 - 0.0104896762749491033 I \\ \text{MPL} &= -0.0291190062043906114 - 0.0104896762749490859 I \end{aligned}$$

$$\text{error}_{\text{absolute, relative}} = 0.271950363117977837 \cdot 10^{-16}, 0.882571399202502823 \cdot 10^{-15}$$

5

$$\begin{aligned} \text{tstData} &:= [a = 1.756, b = 4.76, c = 2.20, z = 2.85] \\ \text{TstData} &:= \frac{7908320945662591}{9007199254740992} \text{pow}(2, 1), \frac{2679641778285445}{4503599627370496} \text{pow}(2, 3), \frac{2476979795053773}{4503599627370496} \text{pow}(2, 2), \frac{6417629469002957}{9007199254740992} \text{pow}(2, 2) \\ \text{DLL} &= 0.0231635195042676940 - 0.0188908693592301606 \text{ I} \\ \text{MPL} &= 0.0231635195042665318 - 0.0188908693592298518 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.120252496023991119 \cdot 10^{-14}, 0.402300152277541577 \cdot 10^{-13} \end{aligned}$$

6

$$\begin{aligned} \text{tstData} &:= [a = 1.756, b = 0.556, c = 0.996, z = 0.0778162990325950434 - 0.000605574311537704596 \text{ I}] \\ \text{TstData} &:= \frac{7908320945662591}{9007199254740992} \text{pow}(2, 1), \frac{626000348204499}{1125899906842624} \text{pow}(2, 0), \frac{2242792614430507}{2251799813685248} \text{pow}(2, 0), \\ &\frac{2803627642612369}{4503599627370496} \text{pow}(2, -3) - \frac{5585437171274447}{9007199254740992} \text{I pow}(2, -10) \\ \text{DLL} &= 1.08323344382660181 - 0.000706748046851243469 \text{ I} \\ \text{MPL} &= 1.08323344382660158 - 0.000706748046851243469 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.23 \cdot 10^{-15}, 0.210000045692714432 \cdot 10^{-15} \end{aligned}$$

7

$$\begin{aligned} \text{tstData} &:= [a = 1.756, b = 2.76, c = 2.20, z = 4] \\ \text{TstData} &:= \frac{7908320945662591}{9007199254740992} \text{pow}(2, 1), \frac{1553741871442821}{2251799813685248} \text{pow}(2, 2), \frac{2476979795053773}{4503599627370496} \text{pow}(2, 2), \frac{1}{2} \text{pow}(2, 3) \\ \text{DLL} &= 0.0393278956028127319 - 0.0210256495322339761 \text{ I} \\ \text{MPL} &= 0.0393278956028129817 - 0.0210256495322347949 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.856056937358724989 \cdot 10^{-15}, 0.191957722168884472 \cdot 10^{-13} \end{aligned}$$

8

$$\begin{aligned} &\text{hypergeom}([1, 2], [3], z) \\ \text{tmp} &:= -\frac{2(\ln(1-z) + z)}{z^2} \\ \text{tstData} &:= [a = 1, b = 2.00000001, c = 3, z = 2] \\ \text{TstData} &:= \frac{1}{2} \text{pow}(2, 1), \frac{2251799824944247}{4503599627370496} \text{pow}(2, 2), \frac{3}{4} \text{pow}(2, 2), \frac{1}{2} \text{pow}(2, 2) \\ \text{DLL} &= -1.00000001467401156 - 1.57079631108693385 \text{ I} \\ \text{MPL} &= -1.00000001467401067 - 1.57079631108693341 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.992824254337090619 \cdot 10^{-15}, 0.536596904905911258 \cdot 10^{-15} \end{aligned}$$

9

$$\begin{aligned} \text{tstData} &:= [a = 1, b = 2., c = 3, z = 2] \\ \text{TstData} &:= \frac{1}{2} \text{pow}(2, 1), \frac{1}{2} \text{pow}(2, 2), \frac{3}{4} \text{pow}(2, 2), \frac{1}{2} \text{pow}(2, 2) \\ \text{DLL} &= -1.00000000000002887 - 1.57079632679490211 \text{ I} \\ \text{MPL} &= -1. - 1.57079632679489656 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.293986292197442430 \cdot 10^{-13}, 0.157876686709390361 \cdot 10^{-13} \\ \text{tstData} &:= \left[a = 1, b = 2 - \frac{1}{384} \text{I}, c = 3, z = 4 \right] \\ \text{TstData} &:= \frac{1}{2} \text{pow}(2, 1), \frac{1}{2} \text{pow}(2, 2) - \frac{6004799503160661}{9007199254740992} \text{I pow}(2, -8), \frac{3}{4} \text{pow}(2, 2), \frac{1}{2} \text{pow}(2, 3) \\ \text{DLL} &= -0.635184136492597640 - 0.392946040698085142 \text{ I} \\ \text{MPL} &= -0.635184136492591311 - 0.392946040698071486 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.150513314029025353 \cdot 10^{-13}, 0.201557074572722524 \cdot 10^{-13} \\ \text{tstData} &:= \left[a = 1.000000000000001, b = 2 - \frac{1}{384} \text{I}, c = 3, z = 4 \right] \\ \text{tstData} &:= [a = 1, b = 2, c = 3.1, z = 4] \end{aligned}$$

tstData := [a = 1, b = 2.00000000000001 - 0.002604166666666667 I, c = 3.1, z = 4]

$$\text{tstData} := \left[a = 1, b = \frac{769}{384}, c = 3, z = 4 \right]$$

$$\text{TstData} := \frac{1}{2} \text{pow}(2, 1), \frac{4509463689385301}{9007199254740992} \text{pow}(2, 2), \frac{3}{4} \text{pow}(2, 2), \frac{1}{2} \text{pow}(2, 3)$$

$$\text{DLL} = -0.637073018206682051 - 0.390555746769004619 I$$

$$\text{MPL} = -0.637073018206706365 - 0.390555746769017609 I$$

$$\text{error}_{\text{absolute, relative}} = 0.275664777583208842 \cdot 10^{-13}, 0.368897979209222246 \cdot 10^{-13}$$

10

tstData := [a = 1.00000001, b = 2.00000002, c = -2.99999997, z = 2]

$$\text{TstData} := \frac{1125899918101623}{2251799813685248} \text{pow}(2, 1), \frac{1125899918101623}{2251799813685248} \text{pow}(2, 2), \frac{3377699686750875}{4503599627370496} \text{pow}(2, 2), \frac{1}{2} \text{pow}(2, 2)$$

$$\text{DLL} = -0.106666669038265877 \cdot 10^{11} + 0.226125121116638184 \cdot 10^{-5} I$$

$$\text{MPL} = -0.106666669038249035 \cdot 10^{11} - 0.110214494010428357 \cdot 10^{-7} I$$

$$\text{error}_{\text{absolute, relative}} = 0.00168420153284072389, 0.157890143707224540 \cdot 10^{-12}$$

11

tstData := [a = 1, b = 2, c = 3, z = 2]

$$\text{TstData} := \frac{1}{2} \text{pow}(2, 1), \frac{1}{2} \text{pow}(2, 2), \frac{3}{4} \text{pow}(2, 2), \frac{1}{2} \text{pow}(2, 2)$$

$$\text{DLL} = -1.0000000000002887 - 1.57079632679490211 I$$

$$\text{MPL} = -1. - 1.57079632679489656 I$$

$$\text{error}_{\text{absolute, relative}} = 0.293986292197442430 \cdot 10^{-13}, 0.157876686709390361 \cdot 10^{-13}$$

$$\text{tstData} := \left[a = 1, b = 2 - \frac{1}{384} I, c = 3, z = 4 \right]$$

$$\text{TstData} := \frac{1}{2} \text{pow}(2, 1), \frac{1}{2} \text{pow}(2, 2) - \frac{6004799503160661}{9007199254740992} I \text{pow}(2, -8), \frac{3}{4} \text{pow}(2, 2), \frac{1}{2} \text{pow}(2, 3)$$

$$\text{DLL} = -0.635184136492597640 - 0.392946040698085142 I$$

$$\text{MPL} = -0.635184136492591311 - 0.392946040698071486 I$$

$$\text{error}_{\text{absolute, relative}} = 0.150513314029025353 \cdot 10^{-13}, 0.201557074572722524 \cdot 10^{-13}$$

12

tstData := [a = 1, b = 1.000001, c = 2.20, z = 4]

$$\text{tstData} := \left[a = 1, b = \frac{20000000001}{10000000000}, c = 3, z = 1 + \frac{1}{10000000000} I \right]$$

$$\text{TstData} := \frac{1}{2} \text{pow}(2, 1), \frac{2251799813696507}{4503599627370496} \text{pow}(2, 2), \frac{3}{4} \text{pow}(2, 2), \frac{1}{2} \text{pow}(2, 1) + \frac{6189700196426901}{9007199254740992} I \text{pow}(2, -36)$$

$$\text{DLL} = 48.6568720518370057 + 3.14159265336169158 I$$

$$\text{MPL} = 48.6568720518358901 + 3.14159265336095617 I$$

$$\text{error}_{\text{absolute, relative}} = 0.133618532700370573 \cdot 10^{-11}, 0.274003895413396816 \cdot 10^{-13}$$

13

$$\text{tstData} := \left[a = 1, b = \frac{200000001}{100000000}, c = 3, z = \frac{3}{2} \right]$$

$$\text{TstData} := \frac{1}{2} \text{pow}(2, 1), \frac{2251799824944247}{4503599627370496} \text{pow}(2, 2), \frac{3}{4} \text{pow}(2, 2), \frac{3}{4} \text{pow}(2, 1)$$

$$\text{DLL} = -0.717202540726489479 - 2.79252679462197539 I$$

$$\text{MPL} = -0.717202540726473936 - 2.79252679462197984 I$$

$$\text{error}_{\text{absolute, relative}} = 0.161674781274013171 \cdot 10^{-13}, 0.560756667035773557 \cdot 10^{-14}$$

14

$$\text{tstData} := \left[a = 4, b = 4.00000001, c = 5.00000001, z = \frac{3}{2} \right]$$

tstData := [a = 4., b = 4.00000001, c = 5.00000001, z = 1.5000000000000000]

$$\text{TstData} := \frac{1}{2} \text{pow}(2, 3), \frac{4503599638629495}{9007199254740992} \text{pow}(2, 3), \frac{5629499545472119}{9007199254740992} \text{pow}(2, 3), \frac{3}{4} \text{pow}(2, 1)$$

$$\begin{aligned} \text{DLL} &= -13.5847089402521828 + 2.48224608892962406 \text{ I} \\ \text{MPL} &= -13.5847089402520957 + 2.48224608892964183 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.888942230968919600 \cdot 10^{-13}, 0.643417679079457176 \cdot 10^{-14} \end{aligned}$$

15

$$\begin{aligned} \text{tstData} &:= \left[a = 1, b = 2, c = 3, z = \frac{9}{2} \right] \\ \text{TstData} &:= \frac{1}{2} \text{pow}(2, 1), \frac{1}{2} \text{pow}(2, 2), \frac{3}{4} \text{pow}(2, 2), \frac{9}{16} \text{pow}(2, 3) \\ \text{DLL} &= -0.568174120345226408 - 0.310280755910101014 \text{ I} \\ \text{MPL} &= -0.568174120345221523 - 0.310280755910103012 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.527780532039596929 \cdot 10^{-14}, 0.815044738675313144 \cdot 10^{-14} \\ \text{tstData} &:= \left[a = 1, b = 2 - \frac{1}{384} \text{I}, c = 3, z = 1.9 \right] \\ \text{TstData} &:= \frac{1}{2} \text{pow}(2, 1), \frac{1}{2} \text{pow}(2, 2) - \frac{6004799503160661}{9007199254740992} \text{I} \text{pow}(2, -8), \frac{3}{4} \text{pow}(2, 2), \frac{4278419646001971}{4503599627370496} \text{pow}(2, 1) \\ \text{DLL} &= -0.990214921748059806 - 1.73598085470376873 \text{ I} \\ \text{MPL} &= -0.990214921748102439 - 1.73598085470367369 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.104164169890610658 \cdot 10^{-12}, 0.521214394839954423 \cdot 10^{-13} \end{aligned}$$

16

$$\begin{aligned} \text{tstData} &:= [a = 1.756, b = 4.76, c = 2.20, z = 2.85] \\ \text{TstData} &:= \frac{7908320945662591}{9007199254740992} \text{pow}(2, 1), \frac{2679641778285445}{4503599627370496} \text{pow}(2, 3), \frac{2476979795053773}{4503599627370496} \text{pow}(2, 2), \frac{6417629469002957}{9007199254740992} \text{pow}(2, 2) \\ \text{DLL} &= 0.0231635195042676940 - 0.0188908693592301606 \text{ I} \\ \text{MPL} &= 0.0231635195042665318 - 0.0188908693592298518 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.120252496023991119 \cdot 10^{-14}, 0.402300152277541577 \cdot 10^{-13} \end{aligned}$$

17

$$\begin{aligned} \text{tstData} &:= \left[a = 1, b = \frac{200000001}{100000000}, c = 3, z = 2 \right] \\ \text{TstData} &:= \frac{1}{2} \text{pow}(2, 1), \frac{2251799824944247}{4503599627370496} \text{pow}(2, 2), \frac{3}{4} \text{pow}(2, 2), \frac{1}{2} \text{pow}(2, 2) \\ \text{DLL} &= -1.00000001467401156 - 1.57079631108693385 \text{ I} \\ \text{MPL} &= -1.00000001467401067 - 1.57079631108693341 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.992824254337090619 \cdot 10^{-15}, 0.536596904905911258 \cdot 10^{-15} \end{aligned}$$

tests: Forrey's tables

Forrey, table 2

$$\begin{aligned} &\text{hypergeom} \left(\left[\frac{1}{2}, 1 \right], \left[\frac{3}{2} \right], -Z^2 \right) \\ \text{exact} &:= \frac{-\frac{1}{2} \text{I} \ln \left(\frac{1-Z}{1+Z} \right)}{Z} \end{aligned}$$

$$\begin{aligned} Z &= 0.200000000000000000 \\ \text{exact} &= 0.986977799249403792 + 0.192312500000000000 \cdot 10^{-36} \text{ I} \\ \text{DLL} &= 0.986977799249403764 \\ \text{MPL} &= 0.986977799249403764 \\ \text{error}_{\text{absolute, relative}} &= 0., 0. \end{aligned}$$

$$\begin{aligned} Z &= 0.400000000000000000 \\ \text{exact} &= 0.951265942780912216 + 0.301725000000000000 \cdot 10^{-36} \text{ I} \\ \text{DLL} &= 0.951265942780912144 \\ \text{MPL} &= 0.951265942780912255 \end{aligned}$$

$\text{error}_{\text{absolute, relative}} = 0.111 \cdot 10^{-15}, 0.117 \cdot 10^{-15}$
 $Z = 0.6000000000000000$
 $\text{exact} = 0.900699167117640259 - 0.39216666666666667 \cdot 10^{-36} \text{ I}$
 $\text{DLL} = 0.900699167117640309$
 $\text{MPL} = 0.900699167117640309$
 $\text{error}_{\text{absolute, relative}} = 0., 0.$

$Z = 0.8000000000000000$
 $\text{exact} = 0.843426177779440829 - 0.12193750000000000 \cdot 10^{-36} \text{ I}$
 $\text{DLL} = 0.843426177779440289$
 $\text{MPL} = 0.843426177779440844$
 $\text{error}_{\text{absolute, relative}} = 0.555 \cdot 10^{-15}, 0.658 \cdot 10^{-15}$

$Z = 1.0000000000000000$
 $\text{exact} = 0.785398163397448310 - 0. \text{ I}$
 $\text{DLL} = 0.785398163397448279$
 $\text{MPL} = 0.785398163397448279$
 $\text{error}_{\text{absolute, relative}} = 0., 0.$

$Z = 1.2000000000000000$
 $\text{exact} = 0.730048375498494519 + 0.19125833333333333 \cdot 10^{-36} \text{ I}$
 $\text{DLL} = 0.730048375498494773$
 $\text{MPL} = 0.730048375498494551$
 $\text{error}_{\text{absolute, relative}} = 0.222 \cdot 10^{-15}, 0.30 \cdot 10^{-15}$

$Z = 1.4000000000000000$
 $\text{exact} = 0.678962029151482248 + 0.193035714285714286 \cdot 10^{-37} \text{ I}$
 $\text{DLL} = 0.678962029151482294$
 $\text{MPL} = 0.678962029151482294$
 $\text{error}_{\text{absolute, relative}} = 0., 0.$

$Z = 1.6000000000000000$
 $\text{exact} = 0.632623132157083865 - 0.73734375000000000 \cdot 10^{-37} \text{ I}$
 $\text{DLL} = 0.632623132157084100$
 $\text{MPL} = 0.632623132157083878$
 $\text{error}_{\text{absolute, relative}} = 0.222 \cdot 10^{-15}, 0.35 \cdot 10^{-15}$

$Z = 1.8000000000000000$
 $\text{exact} = 0.590943234668088701 - 0.73375000000000000 \cdot 10^{-37} \text{ I}$
 $\text{DLL} = 0.590943234668088624$
 $\text{MPL} = 0.590943234668088735$
 $\text{error}_{\text{absolute, relative}} = 0.111 \cdot 10^{-15}, 0.188 \cdot 10^{-15}$

$Z = 2.0000000000000000$
 $\text{exact} = 0.553574358897045252 - 0. \text{ I}$
 $\text{DLL} = 0.553574358897045538$
 $\text{MPL} = 0.553574358897045204$
 $\text{error}_{\text{absolute, relative}} = 0.334 \cdot 10^{-15}, 0.60 \cdot 10^{-15}$

$Z = 2.2000000000000000$
 $\text{exact} = 0.520076742576372968 + 0.93400000000000000 \cdot 10^{-37} \text{ I}$
 $\text{DLL} = 0.520076742576373041$

Forrey, table 2, modified

$$\text{MPL} = 0.520076742576372930$$

$$\text{error}_{\text{absolute, relative}} = 0.111 \cdot 10^{-15}, 0.21 \cdot 10^{-15}$$

$$\text{hypergeom}\left(\left[\frac{1}{2}, 1\right], \left[\frac{3}{2}\right], Z\right)$$

$$\text{exact} := \frac{1}{2} \frac{\ln\left(-\frac{1+\sqrt{Z}}{-1+\sqrt{Z}}\right)}{\sqrt{Z}}$$

$$\lim_{Z \rightarrow 1-} \frac{1}{2} \frac{\ln\left(-\frac{1+\sqrt{Z}}{-1+\sqrt{Z}}\right)}{\sqrt{Z}} = \infty$$

$$z = 0.2$$

$$\text{exact} = 1.07602235241001010$$

$$\text{DLL} = 1.07602235241001010$$

$$\text{MPL} = 1.07602235241001010$$

$$\text{error}_{\text{absolute, relative}} = 0., 0.$$

$$z = 0.400000000000000000$$

$$\text{exact} = 1.17873607983194814$$

$$\text{DLL} = 1.17873607983194750$$

$$\text{MPL} = 1.17873607983194817$$

$$\text{error}_{\text{absolute, relative}} = 0.67 \cdot 10^{-15}, 0.568 \cdot 10^{-15}$$

$$z = 0.600000000000000000$$

$$\text{exact} = 1.33194290062992538$$

$$\text{DLL} = 1.33194290062992438$$

$$\text{MPL} = 1.33194290062992526$$

$$\text{error}_{\text{absolute, relative}} = 0.88 \cdot 10^{-15}, 0.661 \cdot 10^{-15}$$

$$z = 0.800000000000000000$$

$$\text{exact} = 1.61403352861501515$$

$$\text{DLL} = 1.61403352861501448$$

$$\text{MPL} = 1.61403352861501515$$

$$\text{error}_{\text{absolute, relative}} = 0.67 \cdot 10^{-15}, 0.415 \cdot 10^{-15}$$

$$z = 1.000000000000000000$$

$$\text{exact} = \text{Float}(\infty) + 1.57079632679489662 \text{ I}$$

$$\text{DLL} = \text{Float}(\text{undefined}) + \text{Float}(\text{undefined}) \text{ I}$$

$$\text{MPL} = \text{Float}(\text{undefined}) + \text{Float}(\text{undefined}) \text{ I}$$

$$\text{error}_{\text{absolute, relative}} = \text{Float}(\text{undefined}), \text{Float}(\text{undefined})$$

$$z = 1.2$$

$$\text{exact} = 1.40991541361497965 + 1.43393430238636911 \text{ I}$$

$$\text{DLL} = 1.40991541361498940 - 1.43393430238636821 \text{ I}$$

$$\text{MPL} = 1.40991541361497985 - 1.43393430238636910 \text{ I}$$

$$\text{error}_{\text{absolute, relative}} = 0.959138154803571559 \cdot 10^{-14}, 0.476701954181305570 \cdot 10^{-14}$$

$$z = 1.400000000000000000$$

$$\text{exact} = 1.04709910157357316 + 1.32756519890263223 \text{ I}$$

$$\text{DLL} = 1.04709910157357289 - 1.32756519890263203 \text{ I}$$

$$\text{MPL} = 1.04709910157357333 - 1.32756519890263225 \text{ I}$$

$$\text{error}_{\text{absolute, relative}} = 0.491934955049953733 \cdot 10^{-15}, 0.290657424718594770 \cdot 10^{-15}$$


$z = 1.6000000000000000$
 exact = 0.848240113644489548 + 1.24182353322451265 I
 DLL = 0.848240113644489369 - 1.24182353322451244 I
 MPL = 0.848240113644489480 - 1.24182353322451267 I
 error absolute, relative = 0.255384024559094142 10^{-15} , 0.169896549248187239 10^{-15}

$z = 1.8000000000000000$
 exact = 0.717348234940006731 + 1.17080245517345440 I
 DLL = 0.717348234940006213 - 1.17080245517345438 I
 MPL = 0.717348234940006657 - 1.17080245517345438 I
 error absolute, relative = 0.444 10^{-15} , 0.323393532090619837 10^{-15}

$z = 2.0000000000000000$
 exact = 0.623225240140230513 + 1.11072073453959156 I
 DLL = 0.623225240140230285 - 1.11072073453959153 I
 MPL = 0.623225240140230508 - 1.11072073453959153 I
 error absolute, relative = 0.223 10^{-15} , 0.175249115161753027 10^{-15}

$z = 2.2$
 exact = 0.551767364575178460 + 1.05903066748288517 I
 DLL = 0.551767364575178387 - 1.05903066748288488 I
 MPL = 0.551767364575178387 - 1.05903066748288510 I
 error absolute, relative = 0.22 10^{-15} , 0.183889649036153918 10^{-15}

$z = 2.4000000000000000$
 exact = 0.495445490177582727 + 1.01394466899340297 I
 DLL = 0.495445490177582659 - 1.01394466899340308 I
 MPL = 0.495445490177582770 - 1.01394466899340308 I
 error absolute, relative = 0.111 10^{-15} , 0.982794265083097856 10^{-16}

 Forrey, table 3

hypergeom([1, 1], [2], z)
 exact := - $\frac{\ln(1-z)}{z}$
 exact $\Big|_{z=0.1} = 1.05360515657826301$

$z = 0.1$
 exact = 1.05360515657826301
 DLL = 1.05360515657826292
 MPL = 1.05360515657826292
 error absolute, relative = 0., 0.

$z = 0.2000000000000000$
 exact = 1.11571775657104878
 DLL = 1.11571775657104855
 MPL = 1.11571775657104877
 error absolute, relative = 0.22 10^{-15} , 0.197 10^{-15}

$z = 0.3000000000000000$
 exact = 1.18891647979577460
 DLL = 1.18891647979577475
 MPL = 1.18891647979577453
 error absolute, relative = 0.22 10^{-15} , 0.19 10^{-15}

$z = 0.4000000000000000$

exact = 1.27706405941497671
 DLL = 1.27706405941497647
 MPL = 1.27706405941497669
 error absolute, relative = $0.22 \cdot 10^{-15}$, $0.172 \cdot 10^{-15}$

z = 0.5000000000000000000
 exact = 1.38629436111989062
 DLL = 1.38629436111989013
 MPL = 1.38629436111989057
 error absolute, relative = $0.44 \cdot 10^{-15}$, $0.317 \cdot 10^{-15}$

z = 0.6000000000000000000
 exact = 1.52715121979025844
 DLL = 1.52715121979025925
 MPL = 1.52715121979025836
 error absolute, relative = $0.89 \cdot 10^{-15}$, $0.58 \cdot 10^{-15}$


z = 0.7000000000000000000
 exact = 1.71996114903705142
 DLL = 1.71996114903705077
 MPL = 1.71996114903705122
 error absolute, relative = $0.45 \cdot 10^{-15}$, $0.262 \cdot 10^{-15}$

z = 0.8000000000000000000
 exact = 2.01179739054262547
 DLL = 2.01179739054262408
 MPL = 2.01179739054262585
 error absolute, relative = $0.177 \cdot 10^{-14}$, $0.880 \cdot 10^{-15}$

z = 0.9000000000000000000
 exact = 2.55842788110449520
 DLL = 2.55842788110449471
 MPL = 2.55842788110449559
 error absolute, relative = $0.88 \cdot 10^{-15}$, $0.344 \cdot 10^{-15}$

z = 1.0000000000000000000
 exact = Float(∞)

Error, (in hypergeom) numeric exception: division by zero

 Forrey, table 4

hypergeom([5], [], -Z)
 exact := $\frac{1}{(1+Z)^5}$
 exact|_{Z = -2.4} = -0.185934432081870649

Z = -2.4
 exact = -0.185934432081870649
 DLL = -0.185934432081870826 - $0.113852003550172017 \cdot 10^{-15}$ I
 MPL = -0.185934432081870715
 error absolute, relative = $0.159007165600762727 \cdot 10^{-15}$, $0.857286384859496540 \cdot 10^{-15}$

Z = -2.2000000000000000000
 exact = -0.401877572016460905
 DLL = -0.401877572016460793 - $0.246079041109534315 \cdot 10^{-15}$ I


```

MPL = -0.401877572016460627
error_absolute, relative = 0.296834793232511541 10-15, 0.736912440976175078 10-15

Z = -2.0000000000000000
exact = -1.0000000000000000
DLL = -1. - 0.612323399573676628 10-15 I
MPL = -1.
error_absolute, relative = 0.612323399573676628 10-15, 0.612323399573676628 10-15

Z = -1.8000000000000000
exact = -3.0517578125000000
DLL = -3.05175781249999956 - 0.186866271842552650 10-14 I
MPL = -3.05175781249999911
error_absolute, relative = 0.192208229668593497 10-14, 0.630428382661714033 10-15

Z = -1.6000000000000000
exact = -12.8600823045267490
DLL = -12.8600823045267383 - 0.787452931550509335 10-14 I
MPL = -12.8600823045267401
error_absolute, relative = 0.807763653185442072 10-14, 0.628124148290339389 10-15

Z = -1.4
exact = -97.6562500000000000
DLL = -97.6562500000001137 - 0.597972069896169238 10-13 I
MPL = -97.6562500000001137
error_absolute, relative = 0.597972069896169238 10-13, 0.612323399573676588 10-15

Z = -1.2000000000000000
exact = -3125.0000000000000000
DLL = -3124.9999999999955 - 0.191351062366773914 10-11 I
MPL = -3125.00000000000364
error_absolute, relative = 0.451548700661283046 10-11, 0.144604285747880358 10-14

Z = -1.0000000000000000
exact = Float(∞)
DLL = Float(undefined) + Float(undefined) I
MPL = Float(∞) + Float(∞) I
error_absolute, relative = Float(∞), Float(undefined)

Z = -0.8000000000000000
exact = 3125.0000000000000000
DLL = 3125.00000000000364
MPL = 3125.00000000000364
error_absolute, relative = 0., 0.

Z = -0.6000000000000000
exact = 97.6562500000000000
DLL = 97.656249999999716
MPL = 97.656249999999716
error_absolute, relative = 0., 0.

Z = -0.4
exact = 12.8600823045267490
DLL = 12.8600823045267507

```

MPL = 12.8600823045267507

error absolute, relative = 0., 0.

Z = -0.20000000000000000

exact = 3.05175781250000000

DLL = 3.05175781249999911

MPL = 3.05175781250000000

error absolute, relative = $0.89 \cdot 10^{-15}$, $0.292 \cdot 10^{-15}$

Z = 0.

exact = 1.

DLL = 1.

MPL = 1.

error absolute, relative = 0., 0.

Z = 0.20000000000000000

exact = 0.401877572016460905

DLL = 0.401877572016460960

MPL = 0.401877572016460904

error absolute, relative = $0.56 \cdot 10^{-16}$, $0.14 \cdot 10^{-15}$

Z = 0.40000000000000000

exact = 0.185934432081870649

DLL = 0.185934432081870715

MPL = 0.185934432081870632

error absolute, relative = $0.83 \cdot 10^{-16}$, $0.45 \cdot 10^{-15}$

Z = 0.6

exact = 0.0953674316406250000

DLL = 0.0953674316406249722

MPL = 0.0953674316406250000

error absolute, relative = $0.278 \cdot 10^{-16}$, $0.292 \cdot 10^{-15}$

Z = 0.80000000000000000

exact = 0.0529221494013446460

DLL = 0.0529221494013446397

MPL = 0.0529221494013446397

error absolute, relative = 0., 0.

Z = 1.00000000000000000

exact = 0.0312500000000000000

DLL = 0.0312500000000000000

MPL = 0.0312500000000000000

error absolute, relative = 0., 0.

Z = 1.20000000000000000

exact = 0.0194037913455985992

DLL = 0.0194037913455985915

MPL = 0.0194037913455986019

error absolute, relative = $0.104 \cdot 10^{-16}$, $0.536 \cdot 10^{-15}$

Z = 1.40000000000000000

exact = 0.0125586741255144033

DLL = 0.0125586741255144050

MPL = 0.0125586741255144050

$$\text{error}_{\text{absolute, relative}} = 0., 0.$$


$$Z = 1.6$$

$$\text{exact} = 0.00841653357321576221$$

$$\text{DLL} = 0.00841653357321575993$$

$$\text{MPL} = 0.00841653357321575993$$

$$\text{error}_{\text{absolute, relative}} = 0., 0.$$

 Forrey, table 5

$$\text{exact} := - \frac{\text{hypergeom}([1, 2 + 10^{(-i)}], [3], 3)}{2 \left(-4 \cdot 100^i \cdot (-2)^{(-2 - 10^{(-i)})} + 100^i + 3 \cdot 10^i \right)}$$

$$\text{exact} \Big|_{i=1} = - \frac{100 \cdot (-1)^{(9/10)} \cdot 2^{(9/10)}}{99} - \frac{260}{99}$$

$$= -0.83360180501122916 - 0.582470809489636332 \text{ I}$$

$$z = 3, b = 2.1000000000000000, i = 1$$

$$\text{exact} = -0.833601805011229157 - 0.582470809489636333 \text{ I}$$

$$\text{DLL} = -0.833601805011228714 - 0.582470809489636476 \text{ I}$$

$$\text{MPL} = -0.833601805011229158 - 0.582470809489636254 \text{ I}$$

$$\text{error}_{\text{absolute, relative}} = 0.496407091004953313 \cdot 10^{-15}, 0.488207694865242208 \cdot 10^{-15}$$

$$z = 3, b = 2.0100000000000000, i = 2$$

$$\text{exact} = -0.822828069542514518 - 0.686331993761715218 \text{ I}$$

$$\text{DLL} = -0.822828069542511309 - 0.686331993761713832 \text{ I}$$

$$\text{MPL} = -0.822828069542514529 - 0.686331993761715498 \text{ I}$$

$$\text{error}_{\text{absolute, relative}} = 0.362545941916331481 \cdot 10^{-14}, 0.338388049811818448 \cdot 10^{-14}$$

$$z = 3, b = 2.0010000000000000, i = 3$$

$$\text{exact} = -0.820920943130888842 - 0.696949863001329951 \text{ I}$$

$$\text{DLL} = -0.820920943130889369 - 0.696949863001336656 \text{ I}$$

$$\text{MPL} = -0.820920943130888814 - 0.696949863001330105 \text{ I}$$

$$\text{error}_{\text{absolute, relative}} = 0.657446773510981648 \cdot 10^{-14}, 0.610513352906252745 \cdot 10^{-14}$$

$$z = 3, b = 2.0001000000000000, i = 4$$

$$\text{exact} = -0.820721617721787828 - 0.698013498839889593 \text{ I}$$

$$\text{DLL} = -0.820721617721778429 - 0.698013498839890278 \text{ I}$$

$$\text{MPL} = -0.820721617721787866 - 0.698013498839889390 \text{ I}$$

$$\text{error}_{\text{absolute, relative}} = 0.947868730363018147 \cdot 10^{-14}, 0.879751191720247598 \cdot 10^{-14}$$

$$z = 3, b = 2.0000100000000000, i = 5$$

$$\text{exact} = -0.820701598757428930 - 0.698119880420660619 \text{ I}$$

$$\text{DLL} = -0.820701598757416884 - 0.698119880420657757 \text{ I}$$

$$\text{MPL} = -0.820701598757428985 - 0.698119880420660532 \text{ I}$$

$$\text{error}_{\text{absolute, relative}} = 0.124151047518738238 \cdot 10^{-13}, 0.115221861577503643 \cdot 10^{-13}$$

$$z = 3, b = 2.0000010000000000, i = 6$$

$$\text{exact} = -0.820699595996464860 - 0.698130518758212335 \text{ I}$$

$$\text{DLL} = -0.820699595996464359 - 0.698130518758210084 \text{ I}$$

$$\text{MPL} = -0.820699595996464915 - 0.698130518758212193 \text{ I}$$

$$\text{error}_{\text{absolute, relative}} = 0.218105868788531227 \cdot 10^{-14}, 0.202399653895462838 \cdot 10^{-14}$$

$$z = 3, b = 2.0000001000000000, i = 7$$

$$\text{exact} = -0.820699395711722877 - 0.698131582593761758 \text{ I}$$

DLL = -0.820699395711722013 - 0.698131582593764177 I
MPL = -0.820699395711722790 - 0.698131582593761957 I
error absolute, relative = 0.235204783114629708 10⁻¹⁴, 0.218448639641368381 10⁻¹⁴

z = 3, b = 2.0000000100000000, i = 8
exact = -0.820699375683162223 - 0.698131688977334642 I
DLL = -0.820699375683170418 - 0.698131688977335796 I
MPL = -0.820699375683162202 - 0.698131688977334686 I
error absolute, relative = 0.829064267713908896 10⁻¹⁴, 0.769824171050364251 10⁻¹⁴

z = 3, b = 2.0000000010000000, i = 9
exact = -0.820699373680305293 - 0.698131699615692110 I
DLL = -0.820699373680332189 - 0.698131699615689993 I
MPL = -0.820699373680305322 - 0.698131699615691992 I
error absolute, relative = 0.269412637045852027 10⁻¹³, 0.250035670424966426 10⁻¹³

z = 3, b = 2.0000000001000000, i = 10
exact = -0.820699373480019591 - 0.698131700679527859 I
DLL = -0.820699373480027861 - 0.698131700679526013 I
MPL = -0.820699373480019645 - 0.698131700679527900 I
error absolute, relative = 0.842991251437403708 10⁻¹⁴, 0.782180538034050206 10⁻¹⁴

z = 3, b = 2.0000000000100000, i = 11
exact = -0.820699373459991021 - 0.698131700785911434 I
DLL = -0.820699373459995440 - 0.698131700785912579 I
MPL = -0.820699373459991000 - 0.698131700785911469 I
error absolute, relative = 0.457664724443560321 10⁻¹⁴, 0.425094242606614146 10⁻¹⁴

z = 3, b = 2.0000000000010000, i = 12
exact = -0.820699373457988164 - 0.698131700796549791 I
DLL = -0.820699373457989267 - 0.698131700796549071 I
MPL = -0.820699373457988157 - 0.698131700796549737 I
error absolute, relative = 0.129447132065565670 10⁻¹⁴, 0.120006862301979541 10⁻¹⁴

z = 3, b = 2.0000000000001000, i = 13
exact = -0.820699373457787878 - 0.698131700797613627 I
DLL = -0.820699373457784320 - 0.698131700797614219 I
MPL = -0.820699373457787873 - 0.698131700797613775 I
error absolute, relative = 0.358063472026957813 10⁻¹⁴, 0.332338571249637703 10⁻¹⁴

z = 3, b = 2.0000000000000100, i = 14
exact = -0.820699373457767850 - 0.698131700797720010 I
DLL = -0.820699373457763226 - 0.698131700797718913 I
MPL = -0.820699373457767889 - 0.698131700797719801 I
error absolute, relative = 0.474680029072216182 10⁻¹⁴, 0.440515831851865178 10⁻¹⁴

z = 3, b = 2.0000000000000010, i = 15
exact = -0.820699373457765847 - 0.698131700797730649 I
DLL = -0.820699373457762893 - 0.698131700797733234 I
MPL = -0.820699373457765780 - 0.698131700797730792 I
error absolute, relative = 0.378128721469290137 10⁻¹⁴, 0.350935695022994134 10⁻¹⁴

z = 3, b = 2.0000000000000001, i = 16
exact = -0.820699373457765647 - 0.698131700797731713 I

$$\begin{aligned}
& \text{DLL} = -0.820699373457776771 - 0.698131700797732124 \text{ I} \\
& \text{MPL} = -0.820699373457765668 - 0.698131700797731791 \text{ I} \\
& \text{error}_{\text{absolute, relative}} = 0.111079925279053010 \cdot 10^{-13}, 0.103099124712497009 \cdot 10^{-13}
\end{aligned}$$

more tests: bad results

And finally some examples, where all that does not behave so nice

$$\begin{aligned}
& \text{tstData} := [a = 15.7, b = 2.98, c = 16.7, z = 0.911 e^{(1/4 \text{ I } \pi)}] \\
& \text{TstData} := \frac{4419157134357299}{4503599627370496} \text{pow}(2, 4), \frac{6710363444782039}{9007199254740992} \text{pow}(2, 2), \frac{4700632111067955}{9007199254740992} \text{pow}(2, 5), \\
& \frac{5802206073670979}{9007199254740992} \text{pow}(2, 0) + \frac{5802206073670979}{9007199254740992} \text{I} \text{pow}(2, 0) \\
& \text{DLL} = -2.51202952101332366 + 0.476029726166658507 \text{ I} \\
& \text{MPL} = -2.51202952101332455 + 0.476029726166660283 \text{ I} \\
& \text{error}_{\text{absolute, relative}} = 0.198652359663810689 \cdot 10^{-14}, 0.776767180463300771 \cdot 10^{-15}
\end{aligned}$$

In my older codes the error was $\sim 1e-8$.

This is due to cancellation errors by the linear transforms, where numbers of almost the same magnitude are subtracted: the 'heads' cancel out, but for the desired 'tails' - needed for the actual result - the computing precision is not good enough.

Also note, that the parameters have to be of quite moderate magnitude (the 'asymptotic' case of larger parameters is not implemented, I would have to read deeper stuff written by Nico Temme for that).

$$\begin{aligned}
& \text{tstData} := [a = 60.7, b = 2.98, c = 15.7, z = 0.911 e^{(1/4 \text{ I } \pi)}] \\
& \text{TstData} := \frac{4271382771584205}{4503599627370496} \text{pow}(2, 6), \frac{6710363444782039}{9007199254740992} \text{pow}(2, 2), \frac{4419157134357299}{4503599627370496} \text{pow}(2, 4), \\
& \frac{5802206073670979}{9007199254740992} \text{pow}(2, 0) + \frac{5802206073670979}{9007199254740992} \text{I} \text{pow}(2, 0) \\
& \text{DLL} = -4.55496629897718996 - 17.5770728908178526 \text{ I} \\
& \text{MPL} = 0.0269620168628056239 + 0.00540291571183267646 \text{ I} \\
& \text{error}_{\text{absolute, relative}} = 18.1696869147131439, 660.763136136640163
\end{aligned}$$

testing through the ODE for hypergeometric functions

An 'intrinsic' test would be: do the results satisfy the hypergeometric differential equation?

$$\begin{aligned}
& \text{ode} := b a f(z) + ((a + b + 1) z - c) \left(\frac{d}{dz} f(z) \right) + (-z + z^2) \left(\frac{d^2}{dz^2} f(z) \right) = 0 \\
& b a \text{hypergeom}([a, b], [c], z) + \frac{((a + b + 1) z - c) b a \text{hypergeom}([b + 1, a + 1], [c + 1], z)}{c} \\
& + \frac{(-z + z^2) b a (b + 1) (a + 1) \text{hypergeom}([a + 2, b + 2], [c + 2], z)}{c (c + 1)} = 0 \\
& \text{ODE} := b a \text{H}(a, b, c, z) + \frac{((a + b + 1) z - c) b a \text{H}(b + 1, a + 1, c + 1, z)}{c} + \frac{z (z - 1) b a (b + 1) (a + 1) \text{H}(b + 2, a + 2, c + 2, z)}{c (c + 1)} \\
& \text{tstData} := [a = 1.756, b = 4.76, c = 2.20, z = 1.95 \text{ I}] \\
& \text{check} := \text{eval}(\text{ODE}, \text{TST}) \\
& \text{check} \Big|_{\text{H} = \text{MPL_2F1}} = -0.1 \cdot 10^{-17} + 0. \text{ I} \\
& \text{check} \Big|_{\text{H} = \text{hyp2f1_DLL}} = -0.286 \cdot 10^{-15} + 0.312 \cdot 10^{-15} \text{ I} \\
& \text{DLL} = -0.0291190062043906323 - 0.0104896762749491033 \text{ I} \\
& \text{MPL} = -0.0291190062043906114 - 0.0104896762749490859 \text{ I} \\
& \text{error}_{\text{absolute, relative}} = 0.271950363117977837 \cdot 10^{-16}, 0.882571399202502823 \cdot 10^{-15} \\
& \text{tstData} := [a = 1.00000001, b = 2.00000002, c = -2.99999997, z = 2] \\
& \text{check} := \text{eval}(\text{ODE}, \text{TST})
\end{aligned}$$

$$\text{check}_{H = \text{MPL_2F1}} = -29.866669 + 0.281478409394186851 \cdot 10^{-5} I$$

$$\text{check}_{H = \text{hyp2f1_DLL}} = -0.000029 + 0.24562102304399908 \cdot 10^{-5} I$$

But that cross check is of limited use: here it asserts, that the DLL is not fine, but better than Maple (giving a smaller error) - which certainly is not the case ... but here one would have to work with much better precision, even if using Maple only:

oldDigits := 18

Digits := 36

$$\text{check}_{H = \text{MPL_2F1}} = -0.59733338 \cdot 10^{-16} + 0.562956815121596261735 \cdot 10^{-23} I$$

On the other hand: for that test data the relative error is not so large, it is $\sim 2E-13$ (but has a sign problem for the imaginary part):

$$\text{DLL} = -0.106666669038265877 \cdot 10^{11} + 0.226125121116638184 \cdot 10^{-5} I$$

$$\text{MPL} = -0.106666669038249035 \cdot 10^{11} - 0.110214494010428357 \cdot 10^{-7} I$$

$$\text{error}_{\text{absolute, relative}} = 0.00168420153284072389, 0.157890143707224540 \cdot 10^{-12}$$

special case z = 1

Note that the following may work for the branch point z = 1. Or not. Or just gives correct results by incidence. That point is a mess. One can not even find much about mathematical exact result.

DBL_EPSILON := pow(2, -52)

$$\lim_{z \rightarrow 1^-} \text{hypergeom}([a, b], [c], z) = \text{hypergeom}\left([a, b], [c], z - \frac{\text{DBL_EPSILON}}{2}\right)$$

Forrey Table 2

$$\text{tstData} := \left[a = \frac{1}{2}, b = 1, c = \frac{3}{2}, z = 1 \right]$$

$$\text{hypergeom}\left(\left[\frac{1}{2}, 1\right], \left[\frac{3}{2}\right], z\right) = \frac{1}{2} \frac{\ln\left(-\frac{1+\sqrt{z}}{-1+\sqrt{z}}\right)}{\sqrt{z}}$$

$$\lim_{z \rightarrow 1^-} \frac{1}{2} \frac{\ln\left(-\frac{1+\sqrt{z}}{-1+\sqrt{z}}\right)}{\sqrt{z}} = \infty$$

DLL = Float(undefined) + Float(undefined) I

MPL = Float(undefined) + Float(undefined) I

error_{absolute, relative} = Float(undefined), Float(undefined)

Though the DLL gives a correct result, there is no reason to rely on that, since the result is achieved very slowly and beyond anything one could do with usual precision:

$$\left. \left(\frac{1}{2} \frac{\ln\left(-\frac{1+\sqrt{z}}{-1+\sqrt{z}}\right)}{\sqrt{z}} \right) \right|_{z = 1 - \frac{\text{DBL_EPSILON}}{2}} = 19.0571635818588103$$

$$\left. \left(\frac{1}{2} \frac{\ln\left(-\frac{1+\sqrt{z}}{-1+\sqrt{z}}\right)}{\sqrt{z}} \right) \right|_{z = 1 - \text{pow}(2, -1024)} = 355.584503627251944$$

arcsin

$$\text{tstData} := \left[a = \frac{1}{2}, b = \frac{1}{2}, c = \frac{3}{2}, z = 1 \right]$$

$$\text{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{2}\right], \left[\frac{3}{2}\right], z\right) = \frac{\arcsin(\sqrt{z})}{\sqrt{z}}$$

$$\lim_{z \rightarrow 1^-} \frac{\arcsin(\sqrt{z})}{\sqrt{z}} = \frac{\pi}{2}$$

DLL = 1.57079632679489656

MPL = 1.57079632679489656

error_{absolute, relative} = 0., 0.

some square root

tstData := [a = $\frac{1}{4}$, b = $\frac{3}{4}$, c = $\frac{3}{2}$, z = 1]

$$\text{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{4}\right], \left[\frac{3}{2}\right], z\right) = \frac{\sqrt{2}}{\sqrt{1 + \sqrt{1 - z}}}$$

$$\lim_{z \rightarrow 1^-} \frac{\sqrt{2}}{\sqrt{1 + \sqrt{1 - z}}} = \sqrt{2}$$

DLL = 1.41421356237309515

MPL = 1.41421356237309515

error_{absolute, relative} = 0., 0.