

Digits := 18

theDLL := "D:_Work\vc2005\hyp2f1_various\hyp2f1_new\release\hyp2f1.dll"

Define the functions MPL_2F1 (which is Maple's version) and hyp2f1_DLL (which is calling the DLL).

For comparing results one may wish to convert numerical inputs (decimals in Maple) to their nearest IEEE equivalent to have the same input. After that pre-processing a test routine can be feed, which gives both of the results and the relative and absolute errors.

+ prepare for testing

First: look at the 'usual critical case': points on the circle line, which are also on the diagonal (fix points at the usual linear transforms):

$$\text{tstData} := \left[a = \frac{1}{2}, b = \frac{1}{3}, c = \frac{1}{6}, z = e^{(1/3)I\pi} \right]$$

$$\text{TstData} := \frac{1}{2} \text{pow}(2, 0), \frac{6004799503160661}{9007199254740992} \text{pow}(2, -1), \frac{6004799503160661}{9007199254740992} \text{pow}(2, -2), \frac{1}{2} \text{pow}(2, 0) + \frac{3900231685776981}{4503599627370496} I \text{pow}(2, 0)$$

$$\text{DLL} = 0.623714907712016986 + 0.958923198106978791 I$$

$$\text{MPL} = 0.623714907712016986 + 0.958923198106978902 I$$

$$\text{error}_{\text{absolute, relative}} = 0.111 \cdot 10^{-15}, 0.967481594953503273 \cdot 10^{-16}$$

- tests

1

$$\text{tstData} := \left[a = -3\pi, b = \sqrt{2}, c = e, z = \frac{1}{e^\pi} + 1 - 0.1 \cdot 10^{-289} I \right]$$

$$\text{TstData} := \frac{2652839157010665}{4503599627370496} \text{pow}(2, 4), \frac{6369051672525773}{9007199254740992} \text{pow}(2, 1), \frac{6121026514868073}{9007199254740992} \text{pow}(2, 2),$$

$$\frac{587277226695055}{1125899906842624} \text{pow}(2, 1) - \frac{3511119404027961}{4503599627370496} I \text{pow}(2, -963)$$

$$\text{DLL} = 0.0560182538639227498 - 0.175207071073657517 \cdot 10^{-15} I$$

$$\text{MPL} = 0.0560182538639227706 - 0.172417988082930188 \cdot 10^{-15} I$$

$$\text{error}_{\text{absolute, relative}} = 0.209861617245546953 \cdot 10^{-16}, 0.374325963579970485 \cdot 10^{-15}$$

2

$$\text{tstData} := [a = 1, b = 1, c = 2.2, z = 1.9]$$

$$\text{TstData} := \frac{1}{2} \text{pow}(2, 1), \frac{1}{2} \text{pow}(2, 1), \frac{2476979795053773}{4503599627370496} \text{pow}(2, 2), \frac{4278419646001971}{4503599627370496} \text{pow}(2, 1)$$

$$\text{DLL} = 0.315767495606587678 - 1.70873729899199933 I$$

$$\text{MPL} = 0.315767495606590731 - 1.70873729899200000 I$$

$$\text{error}_{\text{absolute, relative}} = 0.312565337169686812 \cdot 10^{-14}, 0.179859721959850429 \cdot 10^{-14}$$

3

$$\text{tstData} := [a = 1.756, b = 4.76, c = 2.20, z = 0.85]$$

$$\text{TstData} := \frac{7908320945662591}{9007199254740992} \text{pow}(2, 1), \frac{2679641778285445}{4503599627370496} \text{pow}(2, 3), \frac{2476979795053773}{4503599627370496} \text{pow}(2, 2), \frac{7656119366529843}{9007199254740992} \text{pow}(2, 0)$$

$$\text{DLL} = 2443.22685352899316$$

$$\text{MPL} = 2443.22685352899134$$

$$\text{error}_{\text{absolute, relative}} = 0.182 \cdot 10^{-11}, 0.74 \cdot 10^{-15}$$

4

$$\text{tstData} := [a = 1.756, b = 4.76, c = 2.20, z = 1.95 I]$$

$$\text{TstData} := \frac{7908320945662591}{9007199254740992} \text{pow}(2, 1), \frac{2679641778285445}{4503599627370496} \text{pow}(2, 3), \frac{2476979795053773}{4503599627370496} \text{pow}(2, 2), \frac{8782019273372467}{9007199254740992} I \text{pow}(2, 1)$$

$$\text{DLL} = -0.0291190062043906149 - 0.0104896762749490911 I$$

$$\text{MPL} = -0.0291190062043906114 - 0.0104896762749490859 I$$

$$\text{error}_{\text{absolute, relative}} = 0.626817357768592748 \cdot 10^{-17}, 0.199844141127976281 \cdot 10^{-15}$$

5

$$\begin{aligned} \text{tstData} &:= [a = 1.756, b = 4.76, c = 2.20, z = 2.85] \\ \text{TstData} &:= \frac{7908320945662591}{9007199254740992} \text{pow}(2, 1), \frac{2679641778285445}{4503599627370496} \text{pow}(2, 3), \frac{2476979795053773}{4503599627370496} \text{pow}(2, 2), \frac{6417629469002957}{9007199254740992} \text{pow}(2, 2) \\ \text{DLL} &= 0.0231635195042665942 - 0.0188908693592305978 \text{ I} \\ \text{MPL} &= 0.0231635195042665318 - 0.0188908693592298518 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.748605209706691795 \cdot 10^{-15}, 0.250441299868204634 \cdot 10^{-13} \end{aligned}$$

6

$$\begin{aligned} \text{tstData} &:= [a = 1.756, b = 0.556, c = 0.996, z = 0.0778162990325950434 - 0.000605574311537704596 \text{ I}] \\ \text{TstData} &:= \frac{7908320945662591}{9007199254740992} \text{pow}(2, 1), \frac{626000348204499}{1125899906842624} \text{pow}(2, 0), \frac{2242792614430507}{2251799813685248} \text{pow}(2, 0), \\ &\frac{2803627642612369}{4503599627370496} \text{pow}(2, -3) - \frac{5585437171274447}{9007199254740992} \text{I} \text{pow}(2, -10) \\ \text{DLL} &= 1.08323344382660181 - 0.000706748046851243469 \text{ I} \\ \text{MPL} &= 1.08323344382660158 - 0.000706748046851243469 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.23 \cdot 10^{-15}, 0.210000045692714432 \cdot 10^{-15} \end{aligned}$$

7

$$\begin{aligned} \text{tstData} &:= [a = 1.756, b = 2.76, c = 2.20, z = 4] \\ \text{TstData} &:= \frac{7908320945662591}{9007199254740992} \text{pow}(2, 1), \frac{1553741871442821}{2251799813685248} \text{pow}(2, 2), \frac{2476979795053773}{4503599627370496} \text{pow}(2, 2), \frac{1}{2} \text{pow}(2, 3) \\ \text{DLL} &= 0.0393278956028138560 - 0.0210256495322339171 \text{ I} \\ \text{MPL} &= 0.0393278956028129817 - 0.0210256495322347949 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.123892426322192916 \cdot 10^{-14}, 0.277816326708372289 \cdot 10^{-13} \end{aligned}$$

8

$$\begin{aligned} &\text{hypergeom}([1, 2], [3], z) \\ \text{tmp} &:= -\frac{2(\ln(1-z) + z)}{z^2} \\ \text{tstData} &:= [a = 1, b = 2.00000001, c = 3, z = 2] \\ \text{TstData} &:= \frac{1}{2} \text{pow}(2, 1), \frac{2251799824944247}{4503599627370496} \text{pow}(2, 2), \frac{3}{4} \text{pow}(2, 2), \frac{1}{2} \text{pow}(2, 2) \\ \text{DLL} &= -1.00000001467398358 - 1.57079631108693540 \text{ I} \\ \text{MPL} &= -1.00000001467401067 - 1.57079631108693341 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.271629932076713519 \cdot 10^{-13}, 0.145871986004886922 \cdot 10^{-13} \end{aligned}$$

9

$$\begin{aligned} \text{tstData} &:= [a = 1, b = 2., c = 3, z = 2] \\ \text{TstData} &:= \frac{1}{2} \text{pow}(2, 1), \frac{1}{2} \text{pow}(2, 2), \frac{3}{4} \text{pow}(2, 2), \frac{1}{2} \text{pow}(2, 2) \\ \text{DLL} &= -0.99999999999882205 - 1.57079632679485104 \text{ I} \\ \text{MPL} &= -1. - 1.57079632679489656 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.126284331668659513 \cdot 10^{-12}, 0.678187650599876868 \cdot 10^{-13} \\ \text{tstData} &:= \left[a = 1, b = 2 - \frac{1}{384} \text{I}, c = 3, z = 4 \right] \\ \text{TstData} &:= \frac{1}{2} \text{pow}(2, 1), \frac{1}{2} \text{pow}(2, 2) - \frac{6004799503160661}{9007199254740992} \text{I} \text{pow}(2, -8), \frac{3}{4} \text{pow}(2, 2), \frac{1}{2} \text{pow}(2, 3) \\ \text{DLL} &= -0.635184136492591755 - 0.392946040698045396 \text{ I} \\ \text{MPL} &= -0.635184136492591311 - 0.392946040698071486 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.26093777257337731 \cdot 10^{-13}, 0.349361346344757233 \cdot 10^{-13} \\ \text{tstData} &:= \left[a = 1.000000000000001, b = 2 - \frac{1}{384} \text{I}, c = 3, z = 4 \right] \\ \text{tstData} &:= [a = 1, b = 2, c = 3.1, z = 4] \end{aligned}$$

tstData := [a = 1, b = 2.00000000000001 - 0.0026041666666666667 I, c = 3.1, z = 4]

$$\text{tstData} := \left[a = 1, b = \frac{769}{384}, c = 3, z = 4 \right]$$

$$\text{TstData} := \frac{1}{2} \text{pow}(2, 1), \frac{4509463689385301}{9007199254740992} \text{pow}(2, 2), \frac{3}{4} \text{pow}(2, 2), \frac{1}{2} \text{pow}(2, 3)$$

$$\text{DLL} = -0.637073018206698705 - 0.390555746769017664 I$$

$$\text{MPL} = -0.637073018206706365 - 0.390555746769017609 I$$

$$\text{error}_{\text{absolute, relative}} = 0.766019745176323768 \cdot 10^{-14}, 0.102512195749839566 \cdot 10^{-13}$$

10

tstData := [a = 1.00000001, b = 2.00000002, c = -2.99999997, z = 2]

$$\text{TstData} := \frac{1125899918101623}{2251799813685248} \text{pow}(2, 1), \frac{1125899918101623}{2251799813685248} \text{pow}(2, 2), -\frac{3377699686750875}{4503599627370496} \text{pow}(2, 2), \frac{1}{2} \text{pow}(2, 2)$$

$$\text{DLL} = -0.106666669038266029 \cdot 10^{11} + 0.633299350738525391 \cdot 10^{-7} I$$

$$\text{MPL} = -0.106666669038249035 \cdot 10^{11} - 0.110214494010428357 \cdot 10^{-7} I$$

$$\text{error}_{\text{absolute, relative}} = 0.00169940000162649417, 0.159320000152482625 \cdot 10^{-12}$$

11

tstData := [a = 1, b = 2, c = 3, z = 2]

$$\text{TstData} := \frac{1}{2} \text{pow}(2, 1), \frac{1}{2} \text{pow}(2, 2), \frac{3}{4} \text{pow}(2, 2), \frac{1}{2} \text{pow}(2, 2)$$

$$\text{DLL} = -0.99999999999882205 - 1.57079632679485104 I$$

$$\text{MPL} = -1. - 1.57079632679489656 I$$

$$\text{error}_{\text{absolute, relative}} = 0.126284331668659513 \cdot 10^{-12}, 0.678187650599876868 \cdot 10^{-13}$$

$$\text{tstData} := \left[a = 1, b = 2 - \frac{1}{384} I, c = 3, z = 4 \right]$$

$$\text{TstData} := \frac{1}{2} \text{pow}(2, 1), \frac{1}{2} \text{pow}(2, 2) - \frac{6004799503160661}{9007199254740992} I \text{pow}(2, -8), \frac{3}{4} \text{pow}(2, 2), \frac{1}{2} \text{pow}(2, 3)$$

$$\text{DLL} = -0.635184136492591755 - 0.392946040698045396 I$$

$$\text{MPL} = -0.635184136492591311 - 0.392946040698071486 I$$

$$\text{error}_{\text{absolute, relative}} = 0.26093777257337731 \cdot 10^{-13}, 0.349361346344757233 \cdot 10^{-13}$$

12

tstData := [a = 1, b = 1.000001, c = 2.20, z = 4]

$$\text{tstData} := \left[a = 1, b = \frac{200000000001}{100000000000}, c = 3, z = 1 + \frac{1}{100000000000} I \right]$$

$$\text{TstData} := \frac{1}{2} \text{pow}(2, 1), \frac{2251799813696507}{4503599627370496} \text{pow}(2, 2), \frac{3}{4} \text{pow}(2, 2), \frac{1}{2} \text{pow}(2, 1) + \frac{6189700196426901}{9007199254740992} I \text{pow}(2, -36)$$

$$\text{DLL} = 48.6568720518369773 + 3.14159265336164539 I$$

$$\text{MPL} = 48.6568720518358901 + 3.14159265336095617 I$$

$$\text{error}_{\text{absolute, relative}} = 0.128725601509567630 \cdot 10^{-11}, 0.263988532875729433 \cdot 10^{-13}$$

13

$$\text{tstData} := \left[a = 1, b = \frac{200000001}{100000000}, c = 3, z = \frac{3}{2} \right]$$

$$\text{TstData} := \frac{1}{2} \text{pow}(2, 1), \frac{2251799824944247}{4503599627370496} \text{pow}(2, 2), \frac{3}{4} \text{pow}(2, 2), \frac{3}{4} \text{pow}(2, 1)$$

$$\text{DLL} = -0.717202540726496252 - 2.79252679462196207 I$$

$$\text{MPL} = -0.717202540726473936 - 2.79252679462197984 I$$

$$\text{error}_{\text{absolute, relative}} = 0.285267726180162203 \cdot 10^{-13}, 0.989418973168681655 \cdot 10^{-14}$$

14

$$\text{tstData} := \left[a = 4, b = 4.00000001, c = 5.00000001, z = \frac{3}{2} \right]$$

tstData := [a = 4., b = 4.00000001, c = 5.00000001, z = 1.5000000000000000]

$$\text{TstData} := \frac{1}{2} \text{pow}(2, 3), \frac{4503599638629495}{9007199254740992} \text{pow}(2, 3), \frac{5629499545472119}{9007199254740992} \text{pow}(2, 3), \frac{3}{4} \text{pow}(2, 1)$$

$$\begin{aligned} \text{DLL} &= -13.5847089402518062 + 2.48224608892959342 \text{ I} \\ \text{MPL} &= -13.5847089402520957 + 2.48224608892964183 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.293519638354914849 \cdot 10^{-12}, 0.212544049928428792 \cdot 10^{-13} \end{aligned}$$

15

$$\begin{aligned} \text{tstData} &:= \left[a = 1, b = 2, c = 3, z = \frac{9}{2} \right] \\ \text{TstData} &:= \frac{1}{2} \text{pow}(2, 1), \frac{1}{2} \text{pow}(2, 2), \frac{3}{4} \text{pow}(2, 2), \frac{9}{16} \text{pow}(2, 3) \\ \text{DLL} &= -0.568174120345240063 - 0.310280755910104344 \text{ I} \\ \text{MPL} &= -0.568174120345221523 - 0.310280755910103012 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.185877869581077349 \cdot 10^{-13}, 0.287115073413063209 \cdot 10^{-13} \\ \text{tstData} &:= \left[a = 1, b = 2 - \frac{1}{384} \text{I}, c = 3, z = 1.9 \right] \\ \text{TstData} &:= \frac{1}{2} \text{pow}(2, 1), \frac{1}{2} \text{pow}(2, 2) - \frac{6004799503160661}{9007199254740992} \text{I pow}(2, -8), \frac{3}{4} \text{pow}(2, 2), \frac{4278419646001971}{4503599627370496} \text{pow}(2, 1) \\ \text{DLL} &= -0.990214921748110766 - 1.73598085470367036 \text{ I} \\ \text{MPL} &= -0.990214921748102439 - 1.73598085470367369 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.896815638802089579 \cdot 10^{-14}, 0.448776071654352316 \cdot 10^{-14} \end{aligned}$$

16

$$\begin{aligned} \text{tstData} &:= [a = 1.756, b = 4.76, c = 2.20, z = 2.85] \\ \text{TstData} &:= \frac{7908320945662591}{9007199254740992} \text{pow}(2, 1), \frac{2679641778285445}{4503599627370496} \text{pow}(2, 3), \frac{2476979795053773}{4503599627370496} \text{pow}(2, 2), \frac{6417629469002957}{9007199254740992} \text{pow}(2, 2) \\ \text{DLL} &= 0.0231635195042665942 - 0.0188908693592305978 \text{ I} \\ \text{MPL} &= 0.0231635195042665318 - 0.0188908693592298518 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.748605209706691795 \cdot 10^{-15}, 0.250441299868204634 \cdot 10^{-13} \end{aligned}$$

17

$$\begin{aligned} \text{tstData} &:= \left[a = 1, b = \frac{200000001}{100000000}, c = 3, z = 2 \right] \\ \text{TstData} &:= \frac{1}{2} \text{pow}(2, 1), \frac{2251799824944247}{4503599627370496} \text{pow}(2, 2), \frac{3}{4} \text{pow}(2, 2), \frac{1}{2} \text{pow}(2, 2) \\ \text{DLL} &= -1.00000001467398358 - 1.57079631108693540 \text{ I} \\ \text{MPL} &= -1.00000001467401067 - 1.57079631108693341 \text{ I} \\ \text{error}_{\text{absolute, relative}} &= 0.271629932076713519 \cdot 10^{-13}, 0.145871986004886922 \cdot 10^{-13} \end{aligned}$$

tests: Forrey's tables

Forrey, table 2

$$\begin{aligned} &\text{hypergeom} \left(\left[\frac{1}{2}, 1 \right], \left[\frac{3}{2} \right], -Z^2 \right) \\ \text{exact} &:= \frac{-\frac{1}{2} \text{I} \ln \left(\frac{1-Z}{1+Z} \right)}{Z} \end{aligned}$$

$$\begin{aligned} Z &= 0.200000000000000000 \\ \text{exact} &= 0.986977799249403792 + 0.192312500000000000 \cdot 10^{-36} \text{ I} \\ \text{DLL} &= 0.986977799249403764 \\ \text{MPL} &= 0.986977799249403764 \\ \text{error}_{\text{absolute, relative}} &= 0., 0. \end{aligned}$$

$$\begin{aligned} Z &= 0.400000000000000000 \\ \text{exact} &= 0.951265942780912216 + 0.301725000000000000 \cdot 10^{-36} \text{ I} \\ \text{DLL} &= 0.951265942780912144 \\ \text{MPL} &= 0.951265942780912255 \end{aligned}$$

$\text{error}_{\text{absolute, relative}} = 0.111 \cdot 10^{-15}, 0.117 \cdot 10^{-15}$
 $Z = 0.6000000000000000$
 $\text{exact} = 0.900699167117640259 - 0.39216666666666667 \cdot 10^{-36} \text{ I}$
 $\text{DLL} = 0.900699167117640309$
 $\text{MPL} = 0.900699167117640309$
 $\text{error}_{\text{absolute, relative}} = 0., 0.$

$Z = 0.8000000000000000$
 $\text{exact} = 0.843426177779440829 - 0.12193750000000000 \cdot 10^{-36} \text{ I}$
 $\text{DLL} = 0.843426177779440289$
 $\text{MPL} = 0.843426177779440844$
 $\text{error}_{\text{absolute, relative}} = 0.555 \cdot 10^{-15}, 0.658 \cdot 10^{-15}$

$Z = 1.0000000000000000$
 $\text{exact} = 0.785398163397448310 - 0. \text{ I}$
 $\text{DLL} = 0.785398163397448501$
 $\text{MPL} = 0.785398163397448279$
 $\text{error}_{\text{absolute, relative}} = 0.222 \cdot 10^{-15}, 0.28 \cdot 10^{-15}$

$Z = 1.2000000000000000$
 $\text{exact} = 0.730048375498494519 + 0.19125833333333333 \cdot 10^{-36} \text{ I}$
 $\text{DLL} = 0.730048375498494440$
 $\text{MPL} = 0.730048375498494551$
 $\text{error}_{\text{absolute, relative}} = 0.111 \cdot 10^{-15}, 0.152 \cdot 10^{-15}$

$Z = 1.4000000000000000$
 $\text{exact} = 0.678962029151482248 + 0.193035714285714286 \cdot 10^{-37} \text{ I}$
 $\text{DLL} = 0.678962029151482183$
 $\text{MPL} = 0.678962029151482294$
 $\text{error}_{\text{absolute, relative}} = 0.111 \cdot 10^{-15}, 0.163 \cdot 10^{-15}$

$Z = 1.6000000000000000$
 $\text{exact} = 0.632623132157083865 - 0.73734375000000000 \cdot 10^{-37} \text{ I}$
 $\text{DLL} = 0.632623132157083767$
 $\text{MPL} = 0.632623132157083878$
 $\text{error}_{\text{absolute, relative}} = 0.111 \cdot 10^{-15}, 0.175 \cdot 10^{-15}$

$Z = 1.8000000000000000$
 $\text{exact} = 0.590943234668088701 - 0.73375000000000000 \cdot 10^{-37} \text{ I}$
 $\text{DLL} = 0.590943234668088624$
 $\text{MPL} = 0.590943234668088735$
 $\text{error}_{\text{absolute, relative}} = 0.111 \cdot 10^{-15}, 0.188 \cdot 10^{-15}$

$Z = 2.0000000000000000$
 $\text{exact} = 0.553574358897045252 - 0. \text{ I}$
 $\text{DLL} = 0.553574358897045204$
 $\text{MPL} = 0.553574358897045204$
 $\text{error}_{\text{absolute, relative}} = 0., 0.$

$Z = 2.2000000000000000$
 $\text{exact} = 0.520076742576372968 + 0.93400000000000000 \cdot 10^{-37} \text{ I}$

Forrey, table 2, modified

$$\text{DLL} = 0.520076742576372819$$

$$\text{MPL} = 0.520076742576372930$$

$$\text{error}_{\text{absolute, relative}} = 0.111 \cdot 10^{-15}, 0.213 \cdot 10^{-15}$$

$$\text{hypergeom}\left(\left[\frac{1}{2}, 1\right], \left[\frac{3}{2}\right], Z\right)$$

$$\text{exact} := \frac{1}{2} \frac{\ln\left(-\frac{1+\sqrt{Z}}{-1+\sqrt{Z}}\right)}{\sqrt{Z}}$$

$$\lim_{Z \rightarrow 1^-} \frac{1}{2} \frac{\ln\left(-\frac{1+\sqrt{Z}}{-1+\sqrt{Z}}\right)}{\sqrt{Z}} = \infty$$

$$z = 0.2$$

$$\text{exact} = 1.07602235241001010$$

$$\text{DLL} = 1.07602235241001010$$

$$\text{MPL} = 1.07602235241001010$$

$$\text{error}_{\text{absolute, relative}} = 0., 0.$$

$$z = 0.400000000000000000$$

$$\text{exact} = 1.17873607983194814$$

$$\text{DLL} = 1.17873607983194750$$

$$\text{MPL} = 1.17873607983194817$$

$$\text{error}_{\text{absolute, relative}} = 0.67 \cdot 10^{-15}, 0.568 \cdot 10^{-15}$$

$$z = 0.600000000000000000$$

$$\text{exact} = 1.33194290062992538$$

$$\text{DLL} = 1.33194290062992438$$

$$\text{MPL} = 1.33194290062992526$$

$$\text{error}_{\text{absolute, relative}} = 0.88 \cdot 10^{-15}, 0.661 \cdot 10^{-15}$$

$$z = 0.800000000000000000$$

$$\text{exact} = 1.61403352861501515$$

$$\text{DLL} = 1.61403352861501448$$

$$\text{MPL} = 1.61403352861501515$$

$$\text{error}_{\text{absolute, relative}} = 0.67 \cdot 10^{-15}, 0.415 \cdot 10^{-15}$$

$$z = 1.000000000000000000$$

$$\text{exact} = \text{Float}(\infty) + 1.57079632679489662 \text{ I}$$

$$\text{DLL} = 0.$$

$$\text{MPL} = \text{Float}(\text{undefined}) + \text{Float}(\text{undefined}) \text{ I}$$

$$\text{error}_{\text{absolute, relative}} = \text{Float}(\text{undefined}), \text{Float}(\text{undefined})$$

$$z = 1.2$$

$$\text{exact} = 1.40991541361497965 + 1.43393430238636911 \text{ I}$$

$$\text{DLL} = 1.40991541361497696 - 1.43393430238634823 \text{ I}$$

$$\text{MPL} = 1.40991541361497985 - 1.43393430238636910 \text{ I}$$

$$\text{error}_{\text{absolute, relative}} = 0.210691480606122278 \cdot 10^{-13}, 0.104773341940849975 \cdot 10^{-13}$$

$$z = 1.400000000000000000$$

$$\text{exact} = 1.04709910157357316 + 1.32756519890263223 \text{ I}$$

$$\text{DLL} = 1.04709910157357133 - 1.32756519890266933 \text{ I}$$

$$\text{MPL} = 1.04709910157357333 - 1.32756519890263225 \text{ I}$$

$$\text{error}_{\text{absolute, relative}} = 0.371338982602150188 \cdot 10^{-13}, 0.219649099527479326 \cdot 10^{-13}$$


$z = 1.6000000000000000$
 exact = 0.848240113644489548 + 1.24182353322451265 I
 DLL = 0.848240113644492255 - 1.24182353322451355 I
 MPL = 0.848240113644489480 - 1.24182353322451267 I
 error absolute, relative = 0.291118961938242697 10^{-14} , 0.193266828067872007 10^{-14}

$z = 1.8000000000000000$
 exact = 0.717348234940006731 + 1.17080245517345440 I
 DLL = 0.717348234940002771 - 1.17080245517345505 I
 MPL = 0.717348234940006657 - 1.17080245517345438 I
 error absolute, relative = 0.394333564384265926 10^{-14} , 0.287169671956079574 10^{-14}

$z = 2.0000000000000000$
 exact = 0.623225240140230513 + 1.11072073453959156 I
 DLL = 0.623225240140230285 - 1.11072073453959130 I
 MPL = 0.623225240140230508 - 1.11072073453959153 I
 error absolute, relative = 0.320357612676833388 10^{-15} , 0.251370712644621033 10^{-15}

$z = 2.2$
 exact = 0.551767364575178460 + 1.05903066748288517 I
 DLL = 0.551767364575178276 - 1.05903066748288466 I
 MPL = 0.551767364575178387 - 1.05903066748288510 I
 error absolute, relative = 0.453785191472793656 10^{-15} , 0.380278420193566343 10^{-15}

$z = 2.4000000000000000$
 exact = 0.495445490177582727 + 1.01394466899340297 I
 DLL = 0.495445490177582604 - 1.01394466899340263 I
 MPL = 0.495445490177582770 - 1.01394466899340308 I
 error absolute, relative = 0.479641532813829624 10^{-15} , 0.425169888650758196 10^{-15}

 Forrey, table 3

hypergeom([1, 1], [2], z)
 exact := $-\frac{\ln(1-z)}{z}$
 exact $\Big|_{z=0.1} = 1.05360515657826301$

$z = 0.1$
 exact = 1.05360515657826301
 DLL = 1.05360515657826292
 MPL = 1.05360515657826292
 error absolute, relative = 0., 0.

$z = 0.2000000000000000$
 exact = 1.11571775657104878
 DLL = 1.11571775657104855
 MPL = 1.11571775657104877
 error absolute, relative = 0.22 10^{-15} , 0.197 10^{-15}

$z = 0.3000000000000000$
 exact = 1.18891647979577460
 DLL = 1.18891647979577475
 MPL = 1.18891647979577453
 error absolute, relative = 0.22 10^{-15} , 0.19 10^{-15}

$z = 0.400000000000000000$
 exact = 1.27706405941497671
 DLL = 1.27706405941497647
 MPL = 1.27706405941497669
 error absolute, relative = $0.22 \cdot 10^{-15}$, $0.172 \cdot 10^{-15}$

$z = 0.500000000000000000$
 exact = 1.38629436111989062
 DLL = 1.38629436111989013
 MPL = 1.38629436111989057
 error absolute, relative = $0.44 \cdot 10^{-15}$, $0.317 \cdot 10^{-15}$

$z = 0.600000000000000000$
 exact = 1.52715121979025844
 DLL = 1.52715121979025925
 MPL = 1.52715121979025836
 error absolute, relative = $0.89 \cdot 10^{-15}$, $0.58 \cdot 10^{-15}$


$z = 0.700000000000000000$
 exact = 1.71996114903705142
 DLL = 1.71996114903705077
 MPL = 1.71996114903705122
 error absolute, relative = $0.45 \cdot 10^{-15}$, $0.262 \cdot 10^{-15}$

$z = 0.800000000000000000$
 exact = 2.01179739054262547
 DLL = 2.01179739054262408
 MPL = 2.01179739054262585
 error absolute, relative = $0.177 \cdot 10^{-14}$, $0.880 \cdot 10^{-15}$

$z = 0.900000000000000000$
 exact = 2.55842788110449520
 DLL = 2.55842788110449471
 MPL = 2.55842788110449559
 error absolute, relative = $0.88 \cdot 10^{-15}$, $0.344 \cdot 10^{-15}$

$z = 1.000000000000000000$
 exact = Float(∞)

Error, (in hypergeom) numeric exception: division by zero

 Forrey, table 4

$\text{hypergeom}([5], [], -Z)$
 $\text{exact} := \frac{1}{(1+Z)^5}$
 $\text{exact}|_{Z=-2.4} = -0.185934432081870649$

$Z = -2.4$
 exact = -0.185934432081870649
 DLL = -0.185934432081870715 - $0.113852003550171968 \cdot 10^{-15} I$
 MPL = -0.185934432081870715
 error absolute, relative = $0.113852003550171968 \cdot 10^{-15}$, $0.612324216135099867 \cdot 10^{-15}$

$Z = -2.200000000000000000$
 exact = -0.401877572016460905

DLL = -0.401877572016460627 - 0.246079041109534216 10⁻¹⁵ I
MPL = -0.401877572016460627
error_{absolute, relative} = 0.246079041109534216 10⁻¹⁵, 0.612324216135099803 10⁻¹⁵

Z = -2.0000000000000000
exact = -1.0000000000000000
DLL = -1. - 0.612323399573676628 10⁻¹⁵ I
MPL = -1.
error_{absolute, relative} = 0.612323399573676628 10⁻¹⁵, 0.612323399573676628 10⁻¹⁵

Z = -1.8000000000000000
exact = -3.0517578125000000
DLL = -3.05175781249999911 - 0.186866271842552611 10⁻¹⁴ I
MPL = -3.05175781249999911
error_{absolute, relative} = 0.186866271842552611 10⁻¹⁴, 0.612323399573676575 10⁻¹⁵

Z = -1.6000000000000000
exact = -12.8600823045267490
DLL = -12.8600823045267401 - 0.787452931550509493 10⁻¹⁴ I
MPL = -12.8600823045267401
error_{absolute, relative} = 0.787452931550509493 10⁻¹⁴, 0.612324216135099803 10⁻¹⁵

Z = -1.4
exact = -97.6562500000000000
DLL = -97.6562500000001137 - 0.597972069896169238 10⁻¹³ I
MPL = -97.6562500000001137
error_{absolute, relative} = 0.597972069896169238 10⁻¹³, 0.612323399573676588 10⁻¹⁵

Z = -1.2000000000000000
exact = -3125.0000000000000000
DLL = -3125.000000000000364 - 0.191351062366774156 10⁻¹¹ I
MPL = -3125.000000000000364
error_{absolute, relative} = 0.191351062366774156 10⁻¹¹, 0.612324216135099784 10⁻¹⁵

Z = -1.0000000000000000
exact = Float(∞)
DLL = Float(undefined) + Float(undefined) I
MPL = Float(∞) + Float(∞) I
error_{absolute, relative} = Float(∞), Float(undefined)

Z = -0.8000000000000000
exact = 3125.0000000000000000
DLL = 3125.000000000000364
MPL = 3125.000000000000364
error_{absolute, relative} = 0., 0.

Z = -0.6000000000000000
exact = 97.6562500000000000
DLL = 97.656249999999716
MPL = 97.656249999999716
error_{absolute, relative} = 0., 0.

Z = -0.4

exact = 12.8600823045267490
DLL = 12.8600823045267507
MPL = 12.8600823045267507
error_{absolute, relative} = 0., 0.

Z = -0.20000000000000000
exact = 3.0517578125000000
DLL = 3.0517578124999911
MPL = 3.0517578125000000
error_{absolute, relative} = $0.89 \cdot 10^{-15}$, $0.292 \cdot 10^{-15}$

Z = 0.
exact = 1.
DLL = 1.
MPL = 1.
error_{absolute, relative} = 0., 0.

Z = 0.20000000000000000
exact = 0.401877572016460905
DLL = 0.401877572016460960
MPL = 0.401877572016460904
error_{absolute, relative} = $0.56 \cdot 10^{-16}$, $0.14 \cdot 10^{-15}$

Z = 0.40000000000000000
exact = 0.185934432081870649
DLL = 0.185934432081870715
MPL = 0.185934432081870632
error_{absolute, relative} = $0.83 \cdot 10^{-16}$, $0.45 \cdot 10^{-15}$

Z = 0.6
exact = 0.0953674316406250000
DLL = 0.0953674316406249722
MPL = 0.0953674316406250000
error_{absolute, relative} = $0.278 \cdot 10^{-16}$, $0.292 \cdot 10^{-15}$

Z = 0.80000000000000000
exact = 0.0529221494013446460
DLL = 0.0529221494013446397
MPL = 0.0529221494013446397
error_{absolute, relative} = 0., 0.

Z = 1.00000000000000000
exact = 0.03125000000000000
DLL = 0.03125000000000000
MPL = 0.03125000000000000
error_{absolute, relative} = 0., 0.

Z = 1.20000000000000000
exact = 0.0194037913455985992
DLL = 0.0194037913455985915
MPL = 0.0194037913455986019
error_{absolute, relative} = $0.104 \cdot 10^{-16}$, $0.536 \cdot 10^{-15}$

Z = 1.40000000000000000
exact = 0.0125586741255144033
DLL = 0.0125586741255144050

$$\text{MPL} = 0.0125586741255144050$$

$$\text{error}_{\text{absolute, relative}} = 0., 0.$$

$$Z = 1.6$$

$$\text{exact} = 0.00841653357321576221$$

$$\text{DLL} = 0.00841653357321575993$$

$$\text{MPL} = 0.00841653357321575993$$

$$\text{error}_{\text{absolute, relative}} = 0., 0.$$

Forrey, table 5

$$\text{exact} := \frac{\text{hypergeom}([1, 2 + 10^{(-i)}], [3], 3)}{2 \left(4 \cdot 100^i \cdot (-2) \cdot 10^{(-i)} - 100^i - 3 \cdot 10^i \right)}$$

$$\text{exact} \Big|_{i=1} = -\frac{100 \cdot (-1)^{(9/10)} \cdot 2^{(9/10)}}{99} - \frac{260}{99}$$

$$= -0.83360180501122916 - 0.582470809489636332 \text{ I}$$

$$z = 3, b = 2.1000000000000000, i = 1$$

$$\text{exact} = -0.833601805011229157 - 0.582470809489636333 \text{ I}$$

$$\text{DLL} = -0.833601805011223940 - 0.582470809489636476 \text{ I}$$

$$\text{MPL} = -0.833601805011229158 - 0.582470809489636254 \text{ I}$$

$$\text{error}_{\text{absolute, relative}} = 0.522272036394827480 \cdot 10^{-14}, 0.513572688840767045 \cdot 10^{-14}$$

$$z = 3, b = 2.0100000000000000, i = 2$$

$$\text{exact} = -0.822828069542514518 - 0.686331993761715218 \text{ I}$$

$$\text{DLL} = -0.822828069542487772 - 0.686331993761715609 \text{ I}$$

$$\text{MPL} = -0.822828069542514529 - 0.686331993761715498 \text{ I}$$

$$\text{error}_{\text{absolute, relative}} = 0.267572302378254390 \cdot 10^{-13}, 0.249718543945825979 \cdot 10^{-13}$$

$$z = 3, b = 2.0010000000000000, i = 3$$

$$\text{exact} = -0.820920943130888842 - 0.696949863001329951 \text{ I}$$

$$\text{DLL} = -0.820920943130840408 - 0.696949863001330328 \text{ I}$$

$$\text{MPL} = -0.820920943130888814 - 0.696949863001330105 \text{ I}$$

$$\text{error}_{\text{absolute, relative}} = 0.484065136629359004 \cdot 10^{-13}, 0.449512688412626743 \cdot 10^{-13}$$

$$z = 3, b = 2.0001000000000000, i = 4$$

$$\text{exact} = -0.820721617721787828 - 0.698013498839889593 \text{ I}$$

$$\text{DLL} = -0.820721617721788976 - 0.698013498839889945 \text{ I}$$

$$\text{MPL} = -0.820721617721787866 - 0.698013498839889390 \text{ I}$$

$$\text{error}_{\text{absolute, relative}} = 0.124101772751238328 \cdot 10^{-14}, 0.115328201561096830 \cdot 10^{-14}$$

$$z = 3, b = 2.0000100000000000, i = 5$$

$$\text{exact} = -0.820701598757428930 - 0.698119880420660619 \text{ I}$$

$$\text{DLL} = -0.820701598757432982 - 0.698119880420659311 \text{ I}$$

$$\text{MPL} = -0.820701598757428985 - 0.698119880420660532 \text{ I}$$

$$\text{error}_{\text{absolute, relative}} = 0.417933607167454290 \cdot 10^{-14}, 0.387812564737209672 \cdot 10^{-14}$$

$$z = 3, b = 2.0000010000000000, i = 6$$

$$\text{exact} = -0.820699595996464860 - 0.698130518758212335 \text{ I}$$

$$\text{DLL} = -0.820699595996460474 - 0.698130518758210972 \text{ I}$$

$$\text{MPL} = -0.820699595996464915 - 0.698130518758212193 \text{ I}$$

$$\text{error}_{\text{absolute, relative}} = 0.460579222284288460 \cdot 10^{-14}, 0.427489535220991395 \cdot 10^{-14}$$

$$z = 3, b = 2.0000001000000000, i = 7$$

exact = -0.820699395711722877 - 0.698131582593761758 I
DLL = -0.820699395711724899 - 0.698131582593759958 I
MPL = -0.820699395711722790 - 0.698131582593761957 I
error absolute, relative = 0.290583585221188982 10^{-14} , 0.269704160278735469 10^{-14}

z = 3, b = 2.0000000100000000, i = 8
exact = -0.820699375683162223 - 0.698131688977334642 I
DLL = -0.820699375683162313 - 0.698131688977332132 I
MPL = -0.820699375683162202 - 0.698131688977334686 I
error absolute, relative = 0.255641096070252367 10^{-14} , 0.237237466374687680 10^{-14}

z = 3, b = 2.0000000010000000, i = 9
exact = -0.820699373680305293 - 0.698131699615692110 I
DLL = -0.820699373680318089 - 0.698131699615687995 I
MPL = -0.820699373680305322 - 0.698131699615691992 I
error absolute, relative = 0.133780528478549523 10^{-13} , 0.124152663557933909 10^{-13}

z = 3, b = 2.0000000001000000, i = 10
exact = -0.820699373480019591 - 0.698131700679527859 I
DLL = -0.820699373480031413 - 0.698131700679526124 I
MPL = -0.820699373480019645 - 0.698131700679527900 I
error absolute, relative = 0.119012604374494721 10^{-13} , 0.110448402198703290 10^{-13}

z = 3, b = 2.0000000000100000, i = 11
exact = -0.820699373459991021 - 0.698131700785911434 I
DLL = -0.820699373459999548 - 0.698131700785910247 I
MPL = -0.820699373459991000 - 0.698131700785911469 I
error absolute, relative = 0.863490521082889479 10^{-14} , 0.801541608591509975 10^{-14}

z = 3, b = 2.0000000000010000, i = 12
exact = -0.820699373457988164 - 0.698131700796549791 I
DLL = -0.820699373458004588 - 0.698131700796551291 I
MPL = -0.820699373457988157 - 0.698131700796549737 I
error absolute, relative = 0.165043229791470089 10^{-13} , 0.153176557850623376 10^{-13}

z = 3, b = 2.0000000000001000, i = 13
exact = -0.820699373457787878 - 0.698131700797613627 I
DLL = -0.820699373457792536 - 0.698131700797610888 I
MPL = -0.820699373457787873 - 0.698131700797613775 I
error absolute, relative = 0.548437216096792614 10^{-14} , 0.508996606679612269 10^{-14}

z = 3, b = 2.0000000000000100, i = 14
exact = -0.820699373457767850 - 0.698131700797720010 I
DLL = -0.820699373457771664 - 0.698131700797718691 I
MPL = -0.820699373457767889 - 0.698131700797719801 I
error absolute, relative = 0.393480939817928157 10^{-14} , 0.365127288316974943 10^{-14}

z = 3, b = 2.0000000000000010, i = 15
exact = -0.820699373457765847 - 0.698131700797730649 I
DLL = -0.820699373457767001 - 0.698131700797730459 I
MPL = -0.820699373457765780 - 0.698131700797730792 I
error absolute, relative = 0.126559472186004316 10^{-14} , 0.117296167186056553 10^{-14}

z = 3, b = 2.0000000000000001, i = 16

exact = -0.820699373457765647 - 0.698131700797731713 I

DLL = -0.820699373457816961 - 0.698131700797739452 I

MPL = -0.820699373457765668 - 0.698131700797731791 I

error
absolute, relative = 0.518619587944767084 10⁻¹³, 0.481353445232463086 10⁻¹³

more tests: bad results

And finally some examples, where all that does not behave so nice

tstData := [a = 15.7, b = 2.98, c = 16.7, z = 0.911 e^(1 / 4 I π)]

$$\text{TstData} := \frac{4419157134357299}{4503599627370496} \text{pow}(2, 4), \frac{6710363444782039}{9007199254740992} \text{pow}(2, 2), \frac{4700632111067955}{9007199254740992} \text{pow}(2, 5),$$
$$\frac{5802206073670979}{9007199254740992} \text{pow}(2, 0) + \frac{5802206073670979}{9007199254740992} I \text{pow}(2, 0)$$

DLL = -2.51202952101332500 + 0.476029726166658285 I
MPL = -2.51202952101332455 + 0.476029726166660283 I

error
absolute, relative = 0.204804882754293727 10⁻¹⁴, 0.801132672316328146 10⁻¹⁵

In my older codes the error was ~ 1e-8.

This is due to cancellation errors by the linear transforms, where numbers of almost the same magnitude are subtracted: the 'heads' cancel out, but for the desired 'tails' - needed for the actual result - the computing precision is not good enough.

Also note, that the parameters have to be of quite moderate magnitude (the 'asymptotic' case of larger parameters is not implemented, I would have to read deeper stuff written by Nico Temme for that).

tstData := [a = 60.7, b = 2.98, c = 15.7, z = 0.911 e^(1 / 4 I π)]

$$\text{TstData} := \frac{4271382771584205}{4503599627370496} \text{pow}(2, 6), \frac{6710363444782039}{9007199254740992} \text{pow}(2, 2), \frac{4419157134357299}{4503599627370496} \text{pow}(2, 4),$$
$$\frac{5802206073670979}{9007199254740992} \text{pow}(2, 0) + \frac{5802206073670979}{9007199254740992} I \text{pow}(2, 0)$$

DLL = -4.55496629897721039 - 17.5770728908179308 I
MPL = 0.0269620168628056239 + 0.00540291571183267646 I

error
absolute, relative = 18.1696869147132247, 660.763136136643103

testing through the ODE for hypergeometric functions

An 'intrinsic' test would be: do the results satisfy the hypergeometric differential equation?

$$\text{ode} := b a f(z) + ((1 + a + b) z - c) \left(\frac{d}{dz} f(z) \right) + (-z + z^2) \left(\frac{d^2}{dz^2} f(z) \right) = 0$$
$$b a \text{hypergeom}([a, b], [c], z) + \frac{((1 + a + b) z - c) b a \text{hypergeom}([1 + b, a + 1], [c + 1], z)}{c}$$
$$+ \frac{(-z + z^2) b a (1 + b) (a + 1) \text{hypergeom}([2 + b, a + 2], [c + 2], z)}{c (c + 1)} = 0$$
$$\text{ODE} := b a H(a, b, c, z) + \frac{((1 + a + b) z - c) b a H(1 + b, a + 1, c + 1, z)}{c} + \frac{z (z - 1) b a (1 + b) (a + 1) H(2 + b, a + 2, c + 2, z)}{c (c + 1)}$$

tstData := [a = 1.756, b = 4.76, c = 2.20, z = 1.95 I]
check := eval(ODE, TST)

check|_{H = MPL_2F1} = -0.1 10⁻¹⁷ + 0. I
check|_{H = hyp2f1_DLL} = -0.170 10⁻¹⁵ + 0.336 10⁻¹⁵ I

DLL = -0.0291190062043906149 - 0.0104896762749490911 I
MPL = -0.0291190062043906114 - 0.0104896762749490859 I

error
absolute, relative = 0.626817357768592748 10⁻¹⁷, 0.199844141127976281 10⁻¹⁵

tstData := [a = 1.00000001, b = 2.00000002, c = -2.99999997, z = 2]

check := eval(ODE, TST)

$$\text{check}_{\text{H=MPL_2F1}} = -29.866669 + 0.281478409394186851 \cdot 10^{-5} \text{ I}$$

$$\text{check}_{\text{H=hyp2f1_DLL}} = 0.000055 - 0.0000123903162708381986 \text{ I}$$

But that cross check is of limited use: here it asserts, that the DLL is not fine, but better than Maple (giving a smaller error) - which certainly is not the case ... but here one would have to work with much better precision, even if using Maple only:

oldDigits := 18

Digits := 36

$$\text{check}_{\text{H=MPL_2F1}} = -0.59733338 \cdot 10^{-16} + 0.562956815121596261735 \cdot 10^{-23} \text{ I}$$

On the other hand: for that test data the relative error is not so large, it is ~ 2E-13 (but has a sign problem for the imaginary part):

$$\text{DLL} = -0.106666669038266029 \cdot 10^{11} + 0.633299350738525391 \cdot 10^{-7} \text{ I}$$

$$\text{MPL} = -0.106666669038249035 \cdot 10^{11} - 0.110214494010428357 \cdot 10^{-7} \text{ I}$$

$$\text{error}_{\text{absolute, relative}} = 0.00169940000162649417, 0.159320000152482625 \cdot 10^{-12}$$

special case z = 1

Note that the following may work for the branch point z = 1. Or not. Or just gives correct results by incidence. That point is a mess. One can not even find much about mathematical exact result.

DBL_EPSILON := pow(2, -52)

$$\lim_{z \rightarrow 1^-} \text{hypergeom}([a, b], [c], z) = \text{hypergeom}\left([a, b], [c], z - \frac{\text{DBL_EPSILON}}{2}\right)$$

Forrey Table 2

$$\text{tstData} := \left[a = \frac{1}{2}, b = 1, c = \frac{3}{2}, z = 1 \right]$$

$$\text{hypergeom}\left(\left[\frac{1}{2}, 1\right], \left[\frac{3}{2}\right], z\right) = \frac{1}{2} \frac{\ln\left(-\frac{1+\sqrt{z}}{-1+\sqrt{z}}\right)}{\sqrt{z}}$$

$$\lim_{z \rightarrow 1^-} \frac{1}{2} \frac{\ln\left(-\frac{1+\sqrt{z}}{-1+\sqrt{z}}\right)}{\sqrt{z}} = \infty$$

DLL = 0.

MPL = Float(undefined) + Float(undefined) I

error_{absolute, relative} = Float(undefined), Float(undefined)

Though the DLL does **not** give correct result, there is no reason to rely on that, since the result is achieved very slowly and beyond anything one could do with usual precision:

$$\left. \left(\frac{1}{2} \frac{\ln\left(-\frac{1+\sqrt{z}}{-1+\sqrt{z}}\right)}{\sqrt{z}} \right) \right|_{z=1 - \frac{\text{DBL_EPSILON}}{2}} = 19.0571635818588103$$

$$\left. \left(\frac{1}{2} \frac{\ln\left(-\frac{1+\sqrt{z}}{-1+\sqrt{z}}\right)}{\sqrt{z}} \right) \right|_{z=1 - \text{pow}(2, -1024)} = 355.584503627251944$$

arcsin

$$\text{tstData} := \left[a = \frac{1}{2}, b = \frac{1}{2}, c = \frac{3}{2}, z = 1 \right]$$

$$\text{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{2}\right], \left[\frac{3}{2}\right], z\right) = \frac{\arcsin(\sqrt{z})}{\sqrt{z}}$$

$$\lim_{z \rightarrow 1^-} \frac{\arcsin(\sqrt{z})}{\sqrt{z}} = \frac{\pi}{2}$$

DLL = 1.57079632679489656

MPL = 1.57079632679489656

error_{absolute, relative} = 0., 0.

some square root

tstData := [a = $\frac{1}{4}$, b = $\frac{3}{4}$, c = $\frac{3}{2}$, z = 1]

$$\text{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{4}\right], \left[\frac{3}{2}\right], z\right) = \frac{\sqrt{2}}{\sqrt{1 + \sqrt{1 - z}}}$$

$$\lim_{z \rightarrow 1^-} \frac{\sqrt{2}}{\sqrt{1 + \sqrt{1 - z}}} = \sqrt{2}$$

DLL = 1.41421356237309515

MPL = 1.41421356237309515

error_{absolute, relative} = 0., 0.